Time-varying formation of double-integrator discrete-time multi-agent systems with switching topology and time-delay

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***Abstract*—Time-varying formation problem for discrete-time multi-agent systems with double-integrator dynamics is investigated in this article, where the transmission delays and switching topology are discussed. The transmission time-delay can be nonuniform and the graph of the topology is not required to include a spanning tree all the time. A time-varying formation problem is defined; the corresponding formation vector and formation reference function are given. Based on the neighboring states, a linear formation protocol is presented. Using the state transformation method and properties of the stochastic matrix, sufficient conditions for the discrete-time multi-agent systems to complete the specified time-varying formation are given. A formation problem of four agents in a two-dimensional surface is simulated to illustrate the effectiveness of the protocol considering delays and switching topology.**

***Index Terms*—discrete-time, multi-agent systems, time-delay, switching topology**

I. Introduction

Formation control research for multi-agent systems has attracted widespread attention in the past few decades. As an important branch of multi-agent cooperative control, formation control can overcome the shortcomings of single agent's insufficient capability through the cooperation of multiple agents. Formation control technology can be applied in many aspects, such as: target enclosing [1], nuclear radiation detecting [2], wireless backbone implementation [3], and target localization [4].

Based on the first-order formation algorithms in the literature, second-order formation algorithms are presented in [5] to handle the formation problem of double-integrator system. The finite-time formation problem is investigated in [6], where nonlinear formation protocols are presented based on nonlinear consensus protocols to ensure that the formation can be completed in the finite time. Time-varying formation problem of general multi-agent systems is investigated in [7]; time-delays are taken into consideration and those are assumed to be constant. The formation problems of a Unmanned Aerial Vehicle (UAV) system with transmission delays are addressed in [8] and the transmission delays can be time-varying. In [9], formation problem of mobile robot systems with switching topology is studied; an improved feedback controller is proposed. However, the system models in [6]–[9] are all based on continuous-time model.

With the development of digital computing systems, increasing researches on the formation control are based on discrete-time system. Compared to continuous-time controller that needs continuous communication, discrete-time controller can effectively reduce the communication pressure because the transfer of control instruction is intermittent. Formation control and trajectory tracking control problems of general discrete-time multi-agent systems (DMAS) are studied in [10], where the formation structure is fixed. In [11], formation control of DMAS with nonlinear dynamics and uncertainties is investigated, where the output feedback is applied and a p-copy internal model is embedded. In [12], an iterative learning approach

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is applied to solve the formation control of DMAS with nonlinear dynamics. In [10]–[12], the external constraints such as transmission delays and switching topology are not discussed. However, the above constraints often appear in the practical applications.

The consensus of double-integrator DMAS with time-delays and changing topology is studied in [13], where the delays is assumed to be time-varying and the associated graphs can have no spanning trees. In [14], consensus of high-order DMAS with switching topology is investigated, where the changing topology is supposed to Markov switching. Consensus of general DMAS with time-delays is addressed in [15], where the input, output, and communication delays are taken in consideration simultaneously. The leader following consensus and leaderless consensus with dynamically changing topology are studied in [16], where the system matrix is assumed to be neutrally stable. As far as we know, the time-varying formation problem of double-integrator DMAS with changing topology and transmission delays is still open.

Time-varying formation problem of double-integrator DMAS with time-delays and switching topology is investigated in this article. First, the formation, which is time-varying, is defined and corresponding formation vector is given. Then a linear time-varying discrete-time formation protocol based on the neighboring state is proposed. Using the state transformation and properties of the stochastic matrix, a theorem that ensures the realization of the time-varying formation is given. Finally, a simulation is shown to illustrate the effectiveness of the theoretical results.

In contrast to the previous works, the innovations of this article are threefold. First, the models of multi-agent systems and protocol are discretized. The discretized formation controller can reduce the communication resource consumption is more close to the practical applications. However, the system models addressed in [6]–[9] are based on continuous-time. Second, the formation vector can be time-varying, which can provide more potential application scenarios. Third, compared with the studies in [9], [14]–[17], both the communication delays and switching topology are taken into account. The time-delays can be nonuniform; topology does not need to include a spanning tree all the time.

The remainder of this paper is organized as follows. Preliminaries of graph and matrix theory are presented and the time-varying formation problem is defined in section II. A linear time-varying discrete-time formation protocol is given and a theorem that ensures the realization of formation is proven in section III. In section IV, a numerical simulation of four agents formation in a two-dimensional surface is performed. Conclusions are drawn in section V.

The following notations are applied in this paper for simplicity. $\mathbb{R}^{M\times N}$ represents the set of real matrices with M rows and N columns. $\mathbf{1}$ represents a vector $[1,1,\cdots,1]^{\mathrm{T}}$ with an appropriate dimension. $\otimes$ is the Kronecker product.

II. Preliminaries and problem description

A. Basic properties of graph theory

Let $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$ denote a weighted directed graph of N nodes with the set of nodes $\mathcal{V}=\{v_1,v_2,\ldots,v_N\}$, the set of edges $\mathcal{E}=\{\mathcal{E}_{ij}=(v_i,v_j),\ \mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}\}$, and a weighted adjacency matrix $\mathcal{A}=(a_{ij})_{N\times N}$. The weighted elements a_{ij} are nonnegative and it is assumed that $a_{ii}=0$, $\forall i\in\{1,2,\ldots,N\}$. A graph $\mathcal{G}$ is called undirected if $a_{ij}=a_{ji}$, $\forall i,j\in\{1,2,\ldots,N\}$. $\mathcal{N}_j=\{v_j\in\mathcal{V},\ \mathcal{E}_{ji}=(v_j,v_i)\in\mathcal{E}\}$ represents the set of neighbor of node v_j. If there exists a series of edges $(v_i,v_{i_1}),(v_{i_1},v_{i_2}),\ldots(v_{i_l},v_j)$ with v_{i_k} $(k=1,2,\ldots,l)$ different nodes of the graph, then it called that there exists a directed path between nodes v_i and v_j. A directed graph $\mathcal{G}$ is said to contain a directed spanning tree if there exists at least one node that has directed paths to all the other nodes. The in-degree matrix $\mathcal{W}$ is defined as $\mathcal{W}=\mathrm{diag}(\deg_{\mathrm{in}}(v_1),\deg_{\mathrm{in}}(v_2),\ldots,\deg_{\mathrm{in}}(v_N))$, where the in-degree of node v_i is $\deg_{\mathrm{in}}(v_i)=\sum_{j=1,j\neq i}^{N}a_{ij}$. The Laplacian matrix L of graph $\mathcal{G}$ is $L=\mathcal{W}-\mathcal{A}$.

Lemma 1. Let $\mathcal{L}$ be the Laplacian matrix of directed graph $\mathcal{G}$, then 0 is an eigenvalue of L and $\mathbf{1}$ is the associated right eigenvector, i.e. $L\mathbf{1}=0$.

The topology considered in this paper is dynamically changing. The topology of the graph $\mathcal{G}$ at time k is denoted by $\mathcal{G}(k)$. Let $\Omega_N=\{\mathcal{G}(k),k\in\mathbb{Z}_+\}$ be the set of all the possible topologies of graph $\mathcal{G}$. Let $L(k)=(l_{ij}(k))_{N\times N}$ denotes the Laplacian matrix of the graph corresponding to the topology $\mathcal{G}(k)$. For all possible $L(k)$, let d_{max} be the largest diagonal entry of $L(k)$. The communication delay among the agents is described by $\tau_{ij}\in\mathbb{Z}_+,i\neq j$, which represents the time-delay from agent j to i. Assume that the communication delays are bounded, namely $\tau_{ij}\leq\tau_{max}$, where τ_{max} is the maximal delay.

Definition 1 (Wolfowitz, 1963)**.** Consider a square matrix $M=(m_{ij})_{n\times n}\in\mathbb{R}^{n\times n}$,

1) M is called a stochastic matrix if all the elements of M are nonnegative and for $\forall i\in\{1,2,\ldots,n\}$, $\sum_{j=1}^{n}m_{ij}=1$.
2) If M is a stochastic matrix and there exists a constant vector $c\in\mathbb{R}^n$ such that $\prod_{j=1}^{+\infty}M^j=\mathbf{1}c^{\mathrm{T}}$, then M is called stochastic indecomposable and aperiodic (SIA) matrix.

B. Definition of time-varying formation

Consider a DMAS with N agents; the dynamic equation of agent i is modeled as:

$$\begin{aligned}p_i((k+1)T)&=p_i(kT)+Tv_i(kT)\\ v_i((k+1)T)&=v_i(kT)+Tu_i(kT)\end{aligned}\tag{1}$$

where $p_i(kT)\in\mathbb{R}$ and $v_i(kT)\in\mathbb{R}$ are the position and velocity of agent i at time kT, respectively. $u_i(kT)\in\mathbb{R}$

is the control input of agent i at time kT, $T \in \mathbb{R}$ is the sample period, and $k \in \mathbb{Z}_+$.

Denote $x_i(kT) = [p_i(kT), v_i(kT)]^{\mathrm{T}} \in \mathbb{R}^2$ and replace kT by k for simplicity. Then the system (1) can be transformed to:

$$x_i(k+1) = Ax_i(k) + Bu_i(k) \tag{2}$$

where $k \in \mathbb{Z}_+$, $i = 1, 2, \ldots, N$, and

$$A = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ T \end{bmatrix}$$

Definition 2. The system (2) realizes consensus if there exists a $f_R(k) \in \mathbb{R}^2$ such that

$$\lim_{k\to+\infty}(x_i(k) - f_R(k)) = 0 \quad \forall i = 1, 2, \ldots, N \tag{3}$$

where $f_R(k)$ is a vector-valued function and is defined as the consensus reference function.

Denote $x(k) = [x_1(k)^{\mathrm{T}}, x_2(k)^{\mathrm{T}}, \cdots, x_N(k)^{\mathrm{T}}]^{\mathrm{T}}$, and let $h(k) = [h_1(k)^{\mathrm{T}}, h_2(k)^{\mathrm{T}}, \cdots, h_N(k)^{\mathrm{T}}]^{\mathrm{T}}$ be the desired formation vector for system (2), where $h_i(k) = [h_{ip}(k), h_{iv}(k)]^{\mathrm{T}} \in \mathbb{R}^2$ is the formation vector for agent i.

Definition 3. The system (2) realizes the time-varying formation $h(k)$ if there exists a $h_R(k) \in \mathbb{R}^2$ such that

$$\lim_{k\to+\infty}(x_i(k) - h_i(k) - h_R(k)) = 0 \quad \forall i = 1, 2, \ldots, N \tag{4}$$

where $h_R(k)$ is a vector-valued function and is defined as the formation reference function.

Remark 1. According to Definitions 2 and 3, one can obtain that when $h_i(k) \equiv 0$, realizations of consensus and time-varying formation $h_i(k)$ for system (2) are equivalent. In such case, the consensus reference function $f_R(k)$ and formation reference function $h_R(k)$ are the same. More generally, the consensus problem can be considered as a particular case of the time-varying formation problem to be dealt with.

Assumption 1. The formation vector $h_i(k) = [h_{ip}(k), h_{iv}(k)]^{\mathrm{T}} \in \mathbb{R}^2$ satisfies the following condition

$$h_{ip}(k+1) = h_{ip}(k) + Th_{iv}(k),\ i = 1, 2, \ldots, N \tag{5}$$

III. Time-varying formation protocol

For system (2) with the nonuniform communication delays and switching topology, in order to achieve the formation defined by vector $h(k)$, the following discrete-time control protocol is proposed:

$$\begin{aligned} u_i(k) =& K_1(x_i(k) - h_i(k)) + h_{ia}(k)) + \\ & K_2\sum_{j\in\mathcal{N}_i(k)} a_{ij}(k)(x_j(k_\tau) - h_j(k_\tau) - x_i(k) + h_i(k)) \end{aligned} \tag{6}$$

with $i \in \{1, 2, \ldots, N\}$, $k_\tau = k - \tau_{ij}$, $K_1 = [\bar{k}_{11}, \bar{k}_{12}], K_2 = [\bar{k}_{21}, \bar{k}_{22}] \in \mathbb{R}^{1\times 2}$ two control parameter matrix, and $h_{ia}(k) = (h_{iv}(k+1) - h_{iv}(k))/T$.

Under the protocol (6), the system (2) can be described as following:

$$\begin{aligned} x_i(k+1) =& (A + BK_1)x_i(k) + B(h_{ia}(k) - K_1h_i(k)) + \\ & BK_2\sum_{j\in\mathcal{N}_i(k)} a_{ij}(k)(x_j(k_\tau) - h_j(k_\tau) - x_i(k) + h_i(k)) \end{aligned} \tag{7}$$

Let $\varepsilon_i(k) = x_i(k) - h_i(k) = [\varepsilon_{ip}(k), \varepsilon_{iv}(k)]^{\mathrm{T}}$, then (7) is transformed to:

$$\begin{aligned} \varepsilon_i(k+1) =& (A + BK_1)\varepsilon_i(k) \\ & + BK_2\sum_{j\in\mathcal{N}_i(k)} a_{ij}(k)(\varepsilon_j(k_\tau) - \varepsilon_i(k)) \\ & + Ah_i(k) + Bh_{ia}(k) - h_i(k+1) \end{aligned} \tag{8}$$

According to Assumption 1, $Ah_i(k) + Bh_{ia}(k) - h_i(k+1) = 0$ can be obtained. Denote $\varepsilon(k) = [\varepsilon_1(k), \varepsilon_2(k), \cdots, \varepsilon_N(k)]^{\mathrm{T}}$, then one can get that

$$\varepsilon(k+1) = \Xi(k)\varepsilon(k) + \sum_{m=0}^{\tau_{max}}(\Upsilon_m(k) \otimes BK_2)\varepsilon(k-j) \tag{9}$$

where $\Upsilon_j(k) \in \mathbb{R}^{N\times N}$, $\Xi(k) = I_N \otimes (A + BK_1) - L_d(k) \otimes BK_2$, and $L_d(k) = \mathrm{diag}(l_{11}(k), l_{22}(k), \cdots, l_{NN}(k))$.

The ij entry of $\Upsilon_m(k)$ is a_{ij} if $m = \tau_{ij}$, is zero if not. According to the definition of $L(k)$, $L(k) = L_d(k) - \sum_{m=0}^{\tau_{max}} \Upsilon_m(k)$ can be obtained.

Let $\bar{\varepsilon}_i(k) = [\varepsilon_{ip}(k), \varepsilon_{ip}(k) + R_K\varepsilon_{iv}(k)]^{\mathrm{T}}$, where $R_K = \bar{k}_{22}/\bar{k}_{21}$. Denote $\bar{\varepsilon}(k) = [\bar{\varepsilon}_1(k)^{\mathrm{T}}, \bar{\varepsilon}_2(k)^{\mathrm{T}}, \cdots, \bar{\varepsilon}_N(k)^{\mathrm{T}}]^{\mathrm{T}}$, one has that $\bar{\varepsilon}(k) = (I_N \otimes P)\varepsilon(k)$, where

$$P = \begin{bmatrix} 1 & 0 \\ 1 & R_K \end{bmatrix},\ P^{-1} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{R_K} & \frac{1}{R_K} \end{bmatrix}$$

Equation (9) can be converted to:

$$\bar{\varepsilon}(k+1) = \bar{\Xi}(k)\bar{\varepsilon}(k) + \sum_{m=0}^{\tau_{max}}(\Upsilon_m(k) \otimes \bar{B})\bar{\varepsilon}(k-j) \tag{10}$$

where $\bar{\Xi}(k) = (I_N \otimes \bar{A} - L_d(k) \otimes \bar{B})$, $\bar{A} = P(A + BK_1)P^{-1}$

$$\bar{A} = \begin{bmatrix} 1 - \frac{T}{R_K} & \frac{T}{R_K} \\ \bar{k}_{11}TR_K - (1 + \bar{k}_{12}R_K)\frac{T}{R_K} & 1 + (1 + \bar{k}_{12}R_K)\frac{T}{R_K} \end{bmatrix}$$

$$\bar{B} = PBK_2P^{-1} = \begin{bmatrix} 0 & 0 \\ 0 & \bar{k}_{22}T \end{bmatrix}$$

Let $\eta(k) = [\bar{\varepsilon}(k)^{\mathrm{T}}, \bar{\varepsilon}(k-1)^{\mathrm{T}}, \cdots, \bar{\varepsilon}(k - \tau_{max})^{\mathrm{T}}]^{\mathrm{T}}$, then (10) can be further converted to:

$$\eta(k+1) = \Gamma(k)\eta(k) \tag{11}$$

with

$$\Gamma(k) = \begin{bmatrix} \Gamma_0(k) & \Gamma_1(k) & \cdots & \Gamma_{\tau_{max}-1}(k) & \Gamma_{\tau_{max}}(k) \\ I_N & 0 & \cdots & 0 & 0 \\ 0 & I_N & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I_N & 0 \end{bmatrix}$$

where $\Gamma_0(k) = \bar{\Xi}(k) + \Upsilon_0(k) \otimes \bar{B}$ and for $i = 1, 2, \ldots, \tau_{max}$, $\Gamma_i(k) = \Upsilon_i(k) \otimes \bar{B}$.

Lemma 2. The system (2) realizes time-varying formation defined by formation vector $h(k)$ if the system (11) is asymptotically stable, namely $\lim_{k\to+\infty} \eta(k)$ exists.

Proof. Since $\eta(k) = [\bar{\varepsilon}(k)^{\mathrm{T}}, \bar{\varepsilon}(k-1)^{\mathrm{T}}, \cdots, \bar{\varepsilon}(k-\tau_{max})^{\mathrm{T}}]^{\mathrm{T}}$, one has that $\lim_{k\to+\infty} \bar{\varepsilon}(k)$ exists if system (11) is asymptotically stable.

Furthermore, $\lim_{k\to+\infty} \varepsilon(k)$ exists because one has that $\varepsilon(k) = (I_N \otimes P^{-1})\bar{\varepsilon}(k)$ with P a nonsingular matrix.

Since $\varepsilon(k) = x(k) - h(k)$, the existence of $\lim_{k\to+\infty}(x(k) - h(k))$ can be obtained, which means that the system (2) realizes time-varying formation defined by formation vector $h(k)$.

□

Two lemmas will be introduced to facilitate the proof of the stability of system (11).

Lemma 3 (Wolfowitz, 1963)**.** Consider a finite set of SIA matrices $\Psi_1, \Psi_2, \cdots, \Psi_m \in \mathbb{R}^{n\times n}$ and for every series of matrices $\Psi_{j_1}, \Psi_{j_2}, \ldots, \Psi_{j_i}$ with $i > 1$, the matrix product $\Psi_{j_i}\Psi_{j_{i-1}}\cdots\Psi_{j_1}$ is a SIA matrix. Then, for every infinite sequence $\Psi_{j_1}, \Psi_{j_2}, \cdots$, a vector $c \in \mathbb{R}^n$ can be founded such that $\prod_{i=1}^{+\infty} \Psi_{j_i} = \mathbf{1}c^{\mathrm{T}}$.

Lemma 4 (Lin, 2009)**.** If $\Gamma(k)$ is a stochastic matrix, for a period of time $[k_1, k_2]$, $k_2 > k_1$, $k_1, k_2 \in \mathbb{Z}_+$, and the union of the graphs $\bigcup_{k=k_1}^{k_2} \mathcal{G}(k)$ has a spanning tree. Then $\prod_{k=k_1}^{k_2} \Gamma(k)$ is a SIA matrix.

Theorem 1. The system (2) can achieve the time-varying formation $h(k)$ if the following conditions are satisfied:

1) $\bar{k}_{21} > 0$, $\bar{k}_{22} > 0$, $\bar{k}_{12} < 0$, $\bar{k}_{11} = 0$
2) $T < R_K$, $1 + \bar{k}_{12}R_K < 0$
3) $1 + (1 + \bar{k}_{12}R_K)\frac{T}{R_K} > d_{max}\bar{k}_{22}T$
4) There exists an infinite series of time $k_0 = 0, k_1, k_2, \cdots$ and for $m, \mu \in \mathbb{Z}_+$, $0 < k_{m+1} - k_m \le \mu$, the union of graph $\bigcup_{k=k_m}^{k_{m+1}-1} \mathcal{G}(k)$ has a spanning tree.

Proof. From conditions 1) 2) and 3), one can get that all the entries of $\bar{A}$ and $\bar{B}$ are nonnegative and row sum of $\bar{A}$ and $\bar{B}$ are 1, so $\bar{A}$ and $\bar{B}$ are stochastic matrices.

Since $L(k) = L_d(k) - \sum_{m=0}^{\tau_{max}} \Upsilon_m(k)$ and $L\mathbf{1} = 0$, one can get that $\Gamma(k)\mathbf{1} = \mathbf{1}$. Furthermore, from the definition of $\Gamma(k)$, it is clear that all the entries of $\Gamma(k)$ are nonnegative. Thus, $\Gamma(k)$ is a stochastic matrix.

Denote $m_k \in \mathbb{Z}_+$ the largest integer such that $k_{m_k} \le k$ for every $k \ge 0$. Let $\Theta(m) = \Gamma(k_{m+1}-1)\Gamma(k_{m+1}-2)\cdots\Gamma(k_m)$, then,

$$\eta(k+1) = \Gamma(k)\cdots\Gamma(k_{m_k})\prod_{m=0}^{m_k-1}\Theta(m)\eta(0)$$

Since $m, \mu \in \mathbb{Z}_+$, $0 < k_{m+1} - k_m \le \mu$, the union of graph $\bigcup_{k=k_m}^{k_{m+1}-1} \mathcal{G}(k)$ has a spanning tree, so $\Theta(m)$ is a SIA matrix according to Lemma 4. In addition, for an integer j, the union of graph $\bigcup_{i=k_m}^{k_{m+j}-1} \mathcal{G}(i)$ has a spanning tree, thus $\prod_{s=m}^{m+j+1} \Theta(s)$ is a SIA matrix.

All the possible topologies $\mathcal{G}(k)$ form a finite set Ω_N, then all the $a_{ij}(k)$ belong to a finite set. Moreover, $0 < k_{m+1} - k_m \le \mu$, thus one can obtain that all the possible $\Theta(j)$ form also a finite set. Hence, according to Lemma 3, there exists a constant vector $c \in \mathbb{R}^{2(\tau_{max}+1)N}$ such that,

$$\prod_{i=0}^{+\infty}\Theta(i) = \mathbf{1}c^{\mathrm{T}}$$

Thus, it follows that,

$$\begin{aligned}\lim_{k\to+\infty}\eta(k+1) &= \lim_{k\to+\infty}(\Gamma(k)\cdots\Gamma(k_{m_k})\prod_{m=0}^{m_k-1}\Theta(m)\eta(0)) \\ &= \prod_{i=0}^{+\infty}\Theta(i)\eta(0) \\ &= \mathbf{1}c^{\mathrm{T}}\eta(0) \qquad (12)\end{aligned}$$

Then one can obtain that $\lim_{k\to+\infty}\bar{\varepsilon}(k) = \mathbf{1}c^{\mathrm{T}}\eta(0)$, and it follows that $\lim_{k\to+\infty}(x(k) - h(k)) = \lim_{k\to+\infty}\varepsilon(k) = (I_N \otimes P^{-1})\mathbf{1}c^{\mathrm{T}}\eta(0)$. It means that the time-varying formation is realized.

The proof is completed. □

Remark 2. From the proof of Theorem 1, one can see that the time-varying formation can be also achieved if the communication delays among agents are uniform or even reduced to zero. Furthermore, when the topology is fixed, the formation can be realized if the topology has a spanning tree.

Theorem 2. If the system (1) realized the time-varying formation $h(k)$, then the formation reference function $h_R(k)$ satisfies that

$$\lim_{k\to+\infty} h_R(k) = (I_N \otimes P^{-1})\mathbf{1}c^{\mathrm{T}}\eta(0) \qquad (13)$$

where $\eta(0) = \mathbf{1} \otimes [(I_N \otimes P)(x(0) - h(0))]$.

Proof. From the proof of Theorem 1, one obtains that $\lim_{k\to+\infty}\eta(k+1) = \mathbf{1}c^{\mathrm{T}}\eta(0)$. In addition, $\eta(0) = [\bar{\varepsilon}(0)^{\mathrm{T}}, \bar{\varepsilon}(-1)^{\mathrm{T}}, \cdots, \bar{\varepsilon}(-\tau_{max})^{\mathrm{T}}]^{\mathrm{T}}$. Assume that $\bar{\varepsilon}(k \le 0) = \bar{\varepsilon}(0)$, then the following initial equation can be obtained,

$$\eta(0) = \mathbf{1} \otimes \bar{\varepsilon}(0) = \mathbf{1} \otimes [(I_N \otimes P)(x(0) - h(0))] \qquad (14)$$

According to Definition 3, one has that

$$\lim_{k\to+\infty} h_R(k) = \lim_{k\to+\infty}(x_i(k) - h_i(k)) \qquad (15)$$

From the proof of Theorem 1, one can obtain that,

$$\lim_{k\to+\infty}(x(k) - h(k)) = (I_N \otimes P^{-1})\mathbf{1}c^{\mathrm{T}}\eta(0) \qquad (16)$$

Combining (14), (15), and (16), equation (13) can be obtained.

The proof is completed.

□

Remark 3. According to Theorem 2, the formation reference function $h_R(k)$ depends on the initial states of $x(k)$ and $h(k)$. Furthermore, the accurate value of $h_R(k)$ can be obtained if the constant vector c is known.

IV. Simulation

Consider a DMAS with four agents that can move in the $X-Y$ plane. The position and velocity information in the x and y directions are both taken into consideration. The state of each agent is defined as $x_i(k) = [p_{ix}(k), v_{ix}(k), p_{iy}(k), v_{iy}(k)]^{\mathrm{T}}, i \in \{1, 2, 3, 4\}$.

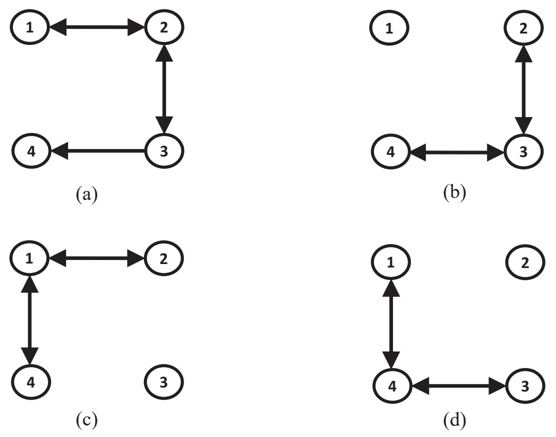

Fig. 1. Four topologies of system.

Sample time T of the simulation is set to 0.1 s. The four topologies are shown in Fig. 1, one can see that the union of graphs $(a) \bigcup (b) \bigcup (c) \bigcup (d)$ has a spanning tree. The topology is cyclically switched in the order of $(a) \rightarrow (b) \rightarrow (c) \rightarrow (d)$ and topology switching interval is set to $5T$.

The communication time-delays among the four agents are set to:

$$\begin{aligned} \tau_{12} = \tau_{21} = \tau_{23} = \tau_{32} = T \\ \tau_{14} = \tau_{41} = \tau_{24} = \tau_{42} = 2T \\ \tau_{13} = \tau_{31} = \tau_{34} = \tau_{43} = 3T \end{aligned} \quad (17)$$

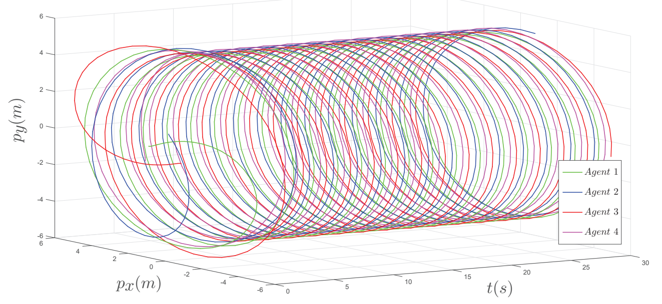

Fig. 2. Position of four agents in $X-Y-t$ space within $t = 30$ s.

The desired formation for the four agents is a circular formation with $r = 5$ m and $\omega = 3.14$ rad/s; the formation vector is,

$$h_i(k) = \begin{bmatrix} rcos(\omega k + (i-1)/2\pi) \\ -\omega rsin(\omega k + (i-1)/2\pi) \\ rsin(\omega k + (i-1)/2\pi) \\ \omega rcos(\omega k + (i-1)/2\pi) \end{bmatrix}, i \in \{1, 2, 3, 4\}$$

According to Theorem 1, the control parameters are chosen as $K_1 = [0, -1]$, $K_2 = [0.2, 0.4]$. The initial states of each agent are set as:

$$\begin{aligned} x_1(0) &= [1, 0, -1, 0.2]^{\mathrm{T}} \\ x_2(0) &= [-1.2, 0.5, 1, 0.5]^{\mathrm{T}} \\ x_3(0) &= [-2, 1, 0.5, 0.6]^{\mathrm{T}} \\ x_4(0) &= [0.5, 0, -1, -0.2]^{\mathrm{T}} \end{aligned}$$

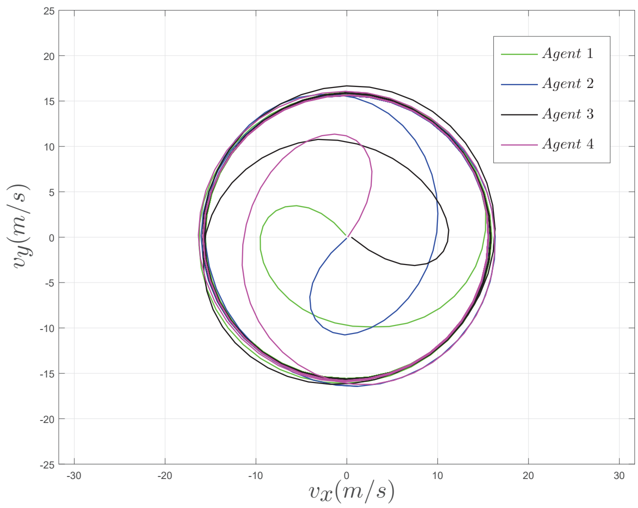

Fig. 3. Velocity of four agents in $X-Y$ plane within $t = 30$ s.

The position evolution of the four agents in $X-Y-t$ space within 30 s is presented in Fig.2. The velocity curves of four agents over 30 s are shown in Fig.3 and the position snapshot of four agents at 30 s is drawn in Fig.4.

It can be obtained from Fig.2 that after the start of the simulation, a circular formation is formed among the four agents. In addition, from Fig.3, the velocity of formation can be obtained and it is about 15.5 m/s. Furthermore, it can be obtained that the four agents form a circle with $r = 5$ m from Fig.4. Therefore, the angular velocity ω is about 3.14 rad/s; it means that the desired formation $h(k)$ is formed.

V. Conclusion

Time-varying formation problem of double-integrator DMAS with time-delays and switching topology was addressed in this paper. A linear discrete-time formation protocol based on neighboring states information was proposed. By applying the state transformation and

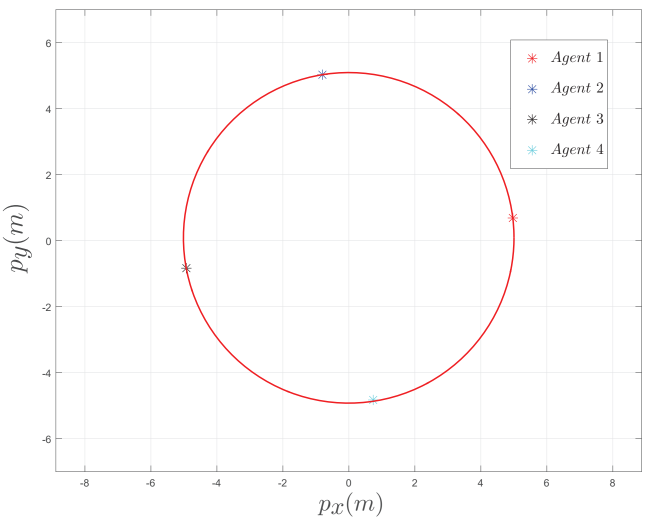

Fig. 4. Position of four agents in $X-Y$ plane at $t=30$ s.

properties of stochastic matrix, sufficient conditions for DMAS with double-integrator dynamics to realize the time-varying formation were given. Simulation results have demonstrated that the proposed formation protocol was effective for the double-integrator DMAS.

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