Discrete Sliding Mode Control for Time-varying Formation Tracking of Multi-UAV System with a Dynamic Leader

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2020 Chinese Automation Congress (CAC) | 978-1-7281-7687-1/20/$31.00 ©2020 IEEE | DOI: 10.1109/CAC51589.2020.9327623

***Abstract*—Time-varying formation tracking control problems for multi-UAV systems are investigated in this paper, where a leader with dynamic input is considered. A discrete-time formation protocol based on sliding mode control method is presented. Using Lyapunov stability approach, sufficient conditions for multi-UAV system to realize the desired formation tracking are given and the quasi-sliding mode band is expressed. An experiment with seven UAVs via Gazebo simulator is performed to verify the effectiveness of the formation tracking protocol.**

***Index Terms*—discrete-time, multi-UAV system, formation tracking, dynamic leader**

I. INTRODUCTION

During the past few years, formation control technologies of multi-agent systems have been extensively investigated and applied in various domains, such as cooperative surveillance [1] and source seeking [2]. With the development of technology, the unmanned aerial vehicle (UAV), as a typical representative of agent, can accomplish more and more missions [3], [4]. To overcome the shortcomings of single UAV in mission, the UAV swarm operations that multiple UAVs cooperate with each other like an entire system become a new trend. For multi-UAV system, how to realize the expected formation by an efficient and robust control method is of theoretical challenges and engineering importance.

Due to great advances in consensus control methods of multi-agent system [5]–[8] over the past years, consensus approach is extended to handle the formation control problem of multi-UAV system. In [9], the formation control problem of a team of Vertical Take-Off and Landing UAVs is investigated, where the communication delay is considered. In [10], an output feedback linearization method is developed to handle the time-varying formation control problem of multi-UAV system. In [11], a consensus based approach is proposed for the time-varying formation control problem of multi-UAV system, where the procedure to design the protocol is given. In [12], necessary and sufficient conditions are given for multi-UAV system with topology changing to realize a time-varying formation.

Note that in [9]–[12], only formation control problems among multiple UAVs are taken into account. In practical applications, just realizing the desired formation is not sufficient, it will be also required to track a reference trajectory given by a leader UAV. In this case, the formation tracking control problems for multi-UAV system arise. In [13], a sliding mode approach is presented to solve the formation tracking control problem of multi-UAV system. In [14], an optimal control approach is developed to deal with the formation tracking control problem of multi-UAV system. In [15], time-varying formation tracking problem of multi-UAV system with switching directed topology is studied, where a protocol that can be designed by four steps is provided to ensure the realization of predefined formation tracking.

It should be pointed out that the above researches are on the basis of continuous model and the proposed formation protocol is also of continuous form. However, with the application of the microcomputers in engineering fields, the discrete-time protocol that can be implemented directly is preferred. Thus, the research of discrete-time formation tracking control problem and the design of discretized protocol are more significant. In [16], discrete-time formation tracking control problems that

This work was supported by the Science and Technology Innovation 2030-Key Project of "New Generation Artificial Intelligence" under Grant 2018AAA0102305, the National Natural Science Foundation of China under Grants 61803014, 61922008, and 61973013, and 61873011, the Beijing Natural Science Foundation under Grant L181003, and the CASIC Foundation under Grant 2019-HT-BH.

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consider the system nonlinearity are investigated. The discrete-time formation control problems for multi-UAV system are studied in [17], where a discrete-time consensus based relative localization approach is developed. However, one common assumption on the above formation tracking control problems for multi-UAV system is that the leader is without any control input, or with known control input. But in actual applications, the above assumption is difficult to be met, especially when the leader is an enemy target to be tracked. As far as we know, the study of the discrete-time formation tracking control problems for multi-UAV system with a dynamic leader is still open.

The discrete-time time-varying formation tracking control problems for discrete multi-UAV system with a dynamic leader are researched in this paper. Sliding surface that is based on the neighboring state information for each UAV of the multi-UAV system is designed. Based on a discrete sliding mode control method, a discrete formation tracking approach is given. The stability of the proposed protocol is proved with the help of Lyapunov stability theory. In addition, the quasi-sliding mode band is obtained. To verify the application of the proposed formation tracking method, a virtual experiment with seven quadrotor UAVs in Gazebo simulator is conducted.

Compared with the previous researches on formation tracking control problems of multi-UAV system, the main contributions of this paper are threefold. First, a leader with maneuvering acceleration in the formation tracking control problem of multi-UAV system is considered, where only the bounds of the maneuvering acceleration need to be known. While in [16], [17], it is assumed that the leader is without any external acceleration or the external acceleration is known to the followers. Second, the UAV model and formation protocol are constructed as discretized form. While the researches in [13]–[15] are based on continuous model, and the proposed protocol cannot be implemented in digital systems directly. Third, a virtual experiment formation tracking platform is constructed and the validity of the proposed protocols are demonstrated by virtual multi-UAV experiments. While in [10], [14], [16], the effectiveness of protocols are verified by traditional numerical simulations, such as MATLAB or Simulink.

Notations: sgn represents the sympolic function; $\mathrm{sgn}(A) = [\mathrm{sgn}(A_1), \mathrm{sgn}(A_2), \ldots, \mathrm{sgn}(A_n)]^{\mathrm{T}}$, if $A = [A_1, A_2, \ldots, A_N] \in \mathbb{R}^n$. In addition, $||A||$ denotes the Euclidean norm for a real vector A. $\mathbf{1}_n$ denotes an n-dimensional column vector with all elements being 1. $\otimes$ indicates the Kronecker product.

II. Preliminaries

Considering a multi-UAV system with one leader labeled 0 and N followers labeled $1, 2, \ldots, N$. The interaction topology among the N followers can be described by a weighted directed graph $\mathcal{G} = (\mathcal{W}, \mathcal{E}, \mathcal{A})$, where $\mathcal{W} = \{w_1, w_2, \ldots, w_N\}$ denotes the set of nodes, $\mathcal{E} = \{e_{ij} = (w_j, w_i), w_i, w_j \in \mathcal{W}\}$ represents the set of edges, and $\mathcal{A} = [a_{ij}]_{N\times N}$ is the weighted adjacency matrix with $i, j \in \{1, 2, \ldots, N\}$. In addition, e_{ij} denotes the edge formed by nodes w_j and w_i, where w_j and w_i are called the parent node and child node, respectively. Moreover, $a_{ij} > 0$ represents the weight of edge e_{ij} if $e_{ij} \in \mathcal{E}$, and $a_{ij} = 0$ if not. Besides, one assumes that $a_{ii} = 0, \forall i = 1, 2, \ldots, N$. A *directed path* between nodes w_i and w_j is defined by a series of edges $(w_i, w_{i1}), (w_{i1}, w_{i2}), \ldots, (w_{il}, w_j)$, where w_{ik} $(k = 1, 2, \ldots, l)$ are different nodes of the graph. For a directed graph $\mathcal{G}$, if there exists at least one node that has directed paths to all the other nodes, then it is said to have a *directed spanning tree*. The in-degree of node w_i is defined as $\deg_{\mathrm{in}}(w_i) = \sum_{j=1, j\neq i}^{N} a_{ij}$. Then the in-degree matrix $\mathcal{D}$ and the Laplacian matrix L are defined as $\mathcal{D} = \mathrm{diag}(\deg_{\mathrm{in}}(w_i), i = 1, 2, \ldots, N)$ and $L = \mathcal{D} - \mathcal{A}$, respectively.

Assume that the communication between the leader and followers is unidirectional, which means that the followers can get the status of the leader, but otherwise it is not. The interaction weight between the leader and follower i is denoted by a_{i0}. $a_{i0} > 0$ if the follower i can get the status of the leader, and $a_{i0} = 0$ if not. In addition, denote $H = \mathrm{diag}(a_{10}, a_{20}, \ldots, a_{N0})$ and $L_H = L + H$.

Lemma 1. *If the directed graph $\mathcal{G}$ contains a directed spanning tree from the leader, then the matrix L_H is invertible.*

III. Problem description

A. Quadrotor UAV modeling

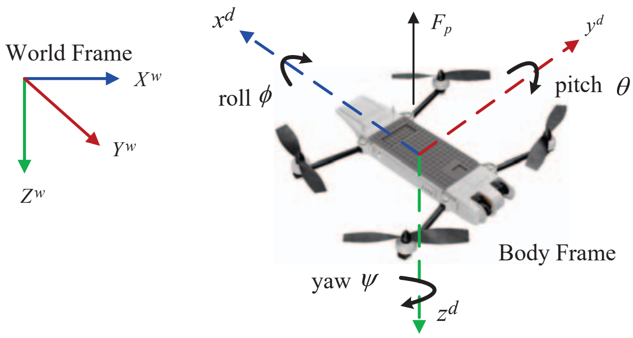

Figure 1: Quadrotor UAV body diagram.

According to Newton's second law, the dynamic equation of the UAV's translational motion can be described as

$$m \begin{bmatrix} \ddot{X}^w \\ \ddot{Y}^w \\ \ddot{Z}^w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} + R_{b2w} \begin{bmatrix} 0 \\ 0 \\ -F_p \end{bmatrix}, \tag{1}$$

where g is the acceleration of gravity, m denotes the mass of the UAV, and $F_p \in \mathbb{R}$ is the combined external force formed by the four propellers. X^w, Y^w, and Z^w are the North, East, and Down positions of UAV in the world frame, respectively. $R_{b2w} \in \mathbb{R}^{3\times 3}$ given in (2) is the transition matrix from the body frame to the world frame, where ϕ, θ, and ψ are the roll, pitch, and yaw angles, respectively.

Decomposing (1) in X and Y directions yields

$$\begin{aligned} m\ddot{X}^w &= -F_p(\sin(\phi)\sin(\psi) + \cos(\phi)\cos(\psi)\sin(\theta)) \\ m\ddot{Y}^w &= F_p(\cos(\psi)\sin(\phi) - \cos(\phi)\sin(\psi)\sin(\theta)) \end{aligned} \tag{3}$$

$$R_{b2w} = \begin{bmatrix} \cos(\theta)\cos(\psi) & \cos(\psi)\sin(\phi)\sin(\theta) - \cos(\phi)\sin(\psi) & \sin(\phi)\sin(\psi) + \cos(\phi)\cos(\psi)\sin(\theta) \\ \cos(\theta)\sin(\psi) & \cos(\phi)\cos(\psi) + \sin(\phi)\sin(\theta)\sin(\psi) & \cos(\phi)\sin(\theta)\sin(\psi) - \cos(\psi)\sin(\phi) \\ -\sin(\theta) & \cos(\theta)\sin(\phi) & \cos(\phi)\cos(\theta) \end{bmatrix} \quad (2)$$

Linearizing (3) according to the following principles:

1) Pitch and roll are small attitude angles. It means that $\sin(\theta) \approx 0$, $\sin(\phi) \approx 0$, $\cos(\theta) \approx 1$, and $\sin(\phi) \approx 1$.
2) Yaw angle does not change. It means that $\psi = \psi_0 = \psi_{des}$, where ψ_0 and ψ_{des} are the initial and desired yaw angles, respectively.
3) UAV is near the hovering steady state, which means that $F_p = mg$.

As a result, one has

$$\begin{aligned} \ddot{X}^w &= -g(\phi\sin(\psi_{des}) + \theta\cos(\psi_{des})), \\ \ddot{Y}^w &= -g(\theta\sin(\psi_{des}) - \phi\cos(\psi_{des})). \end{aligned} \quad (4)$$

Thus, the quadrotor UAV system can be transformed into a linear double integrator model. Furthermore, assume that the desired yaw angle $\psi_{des} = 0$, then the position control in horizontal direction can be decoupled into two separate double integrator models

$$\begin{aligned} \ddot{X}^w &= u_X, \\ \ddot{Y}^w &= u_Y, \end{aligned} \quad (5)$$

where $u_X = -g\theta$, $u_Y = g\phi$ are new control variables for X and Y directions, respectively.

B. Discrete-time multi-UAV system modeling

Each UAV of the multi-UAV system can be reduced as

$$\begin{aligned} \dot{p}_i(t) &= v_i(t), \\ \dot{v}_i(t) &= u_i(t), \end{aligned} \quad (6)$$

where $i = 0, 1, 2 \ldots, N$, $p_i(t) \in \mathbb{R}^n$, $v_i(t) \in \mathbb{R}^n$, and $u_i(t) \in \mathbb{R}^n$ are the position, velocity, and control input of UAV i at time t, respectively.

Substituting the derivative by the forward difference, the double integrator dynamics can be discretized as

$$\begin{aligned} p_i((k+1)T) - p_i(kT) &= Tv_i(kT), \\ v_i((k+1)T) - v_i(kT) &= Tu_i(kT), \end{aligned} \quad (7)$$

where $k \in \mathbb{Z}^+$ and $T > 0$ is the sampling period.

For the sake of clarity and simplicity, let us assume that $n = 1$. However, it should be noted that all the theoretical analysis in the following remains valid for higher dimensional cases, namely, $n \geq 2$. In addition, replace kT by k for simplicity of description.

Denote $x_i(k) = [p_i(k), v_i(k)]^{\mathrm{T}} \in \mathbb{R}^2$, the UAV i can be transformed to the following state space representation

$$x_i(k+1) = Ax_i(k) + Bu_i(k), \quad (8)$$

where

$$A = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ T \end{bmatrix},$$

and $i = 0, 1, 2, \ldots, N$.

Denote $X(k) = [x_1(k)^{\mathrm{T}}, x_2(k)^{\mathrm{T}}, \ldots, x_N(k)^{\mathrm{T}}]^{\mathrm{T}}$ and $U(k) = [u_1(k)^{\mathrm{T}}, u_2(k)^{\mathrm{T}}, \ldots, u_N(k)^{\mathrm{T}}]^{\mathrm{T}}$, then multi-UAV system with N followers can transformed to

$$X(k+1) = (I_N \otimes A)X(k) + (I_N \otimes B)U(k). \quad (9)$$

The leader in the multi-UAV system has the same dynamics as the N followers. The control input $u_0(k)$ of the leader is dynamic but unknown to the followers. Moreover, one assumes that the dynamic input $u_0(k)$ is bounded, which means that there exist two constants u_{min} and u_{max} such that $\forall k \geq 0, u_{min} \leq u_0(k) \leq u_{max}$.

Assumption 1. *For the multi-UAV system* (8)*, there exists a directed spanning tree from the leader UAV.*

C. Problem formulation

Definition 1. *The multi-UAV system* (8) *with one leader and N followers is called to realize a formation tracking, if for $\forall i = 1, 2, \ldots, N$, the following equation is satisfied:*

$$\lim_{k\to\infty} (x_i(k) - f_i(k) - x_0(k)) = \mathbf{0}, \quad (10)$$

where $f_i(k) = [f_{ip}(k), f_{iv}(k)]^{\mathrm{T}}$ is the formation vector defined for follower i with $f_{ip}(k)$ and $f_{iv}(k)$ being the corresponding position and velocity components, respectively.

For sliding mode control approach, the switching function $s_i(k)$ for follower i is defined as

$$\begin{aligned} s_i(k) = K \sum_{j=1}^{N} & a_{ij}(x_i(k) - f_i(k) - x_j(k) + f_j(k)) \\ & + a_{i0}(x_i(k) - f_i(k) - x_0(k)), \end{aligned} \quad (11)$$

where $K \in \mathbb{R}^{1\times 2}$ is a positive coefficient matrix.

Then the global switching function $S(k) = [s_1(k)^{\mathrm{T}}, s_2(k)^{\mathrm{T}}, \ldots, s_N(k)^{\mathrm{T}}]^{\mathrm{T}}$ can be expressed as

$$S(k) = (L_H \otimes K)(X(k) - F(k)) - (H \otimes K)(\mathbf{1}_N \otimes x_0(k)), \quad (12)$$

where $F(k) = [f_1(k)^{\mathrm{T}}, f_2(k)^{\mathrm{T}}, \ldots, f_N(k)^{\mathrm{T}}]^{\mathrm{T}}$.

The control objective of this paper is to design an appropriate discrete-time protocol such that the N follower UAVs can pursue the trajectory $x_0(k)$ of the leader and form the formation $F(k)$ among themselves. The control objective of sliding mode control is to make the sliding mode state of each follower i become $s_i(k) = 0$ and force it to stay there all the time.

IV. Formation tracking protocol design

The discrete formation tracking protocol for multi-UAV system (8) is proposed as

$$\begin{aligned} U(k) = &-\big(L_H \otimes (K\bar{B})\big)^{-1}\Big((L_H \otimes K\bar{A})X(k) \\ &- H_{\mathbf{1}} \otimes (K\bar{A}x_0(k)) - (H \otimes K\bar{B})\tilde{U}_0(k) \\ &+ (qT-1)S(k) + \varepsilon T\,\mathrm{sgn}(S(k)) \\ &- (L_H \otimes K)F(k+1)\Big), \end{aligned} \tag{13}$$

where $\tilde{U}_0(k) = \mathbf{1}_N \otimes \tilde{u}_{i0}(k)$, $\tilde{u}_{i0}(k) = \tilde{u}_1 - \tilde{u}_2\,\mathrm{sgn}(s_i(k))$, $\tilde{u}_1 = (u_{max}+u_{min})/2$, and $\tilde{u}_2 = (u_{max}-u_{min})/2$. ε and q are two control parameters.

Theorem 1. *Under protocol* (13)*, the discrete-time multi-UAV system* (8) *can realize the expected formation tracking if the control parameters satisfy* $\varepsilon > 0$*,* $q > 0$*, and* $1 - qT > 0$*. Furthermore, the quasi-sliding mode band can be obtained*

$$\{s_i(k) : |s_i(k)| \le \Delta\}, \tag{14}$$

where

$$\Delta = \frac{\varepsilon T + a_{i0}K\bar{B}(u_{max} - u_{min})}{2 - qT}. \tag{15}$$

Proof. It holds from (12) that

$$\begin{aligned} S(k+1) = &(L_H \otimes K)(X(k+1) - F(k+1)) \\ &- (H \otimes K)(\mathbf{1}_N \otimes x_0(k+1)). \end{aligned} \tag{16}$$

Replacing $x_0(k+1)$ and $X(k+1)$ by the system equations of leader and followers gives

$$\begin{aligned} S(k+1) =&(L_H \otimes K)((I_N \otimes \bar{A})X(k) + (I_N \otimes \bar{B})U(k)) \\ &- (H \otimes K)\big(\mathbf{1}_N \otimes (\bar{A}x_0(k) + \bar{B}u_0(k))\big) \\ &- (L_H \otimes K)F(k+1). \end{aligned} \tag{17}$$

Substituting protocol (13) into (17) yields

$$\begin{aligned} S(k+1) = &(1-qT)S(k) - H_{\mathbf{1}} \otimes (K\bar{B}u_0(k)) \\ &+ (H \otimes K\bar{B})\tilde{U}_0(k) - \varepsilon T\,\mathrm{sgn}(S(k)). \end{aligned} \tag{18}$$

For each UAV i, one has from (18) that

$$\begin{aligned} s_i(k+1) = &(1-qT)s_i(k) - \varepsilon T\,\mathrm{sgn}(s_i(k)) \\ &- a_{i0}K\bar{B}(u_0(k) - \tilde{u}_{i0}(k)). \end{aligned} \tag{19}$$

Choose the Lyapunov candidate function as

$$V_s(k) = \sum_{i=1}^{N}(s_i(k)^2), \tag{20}$$

which means that

$$\begin{aligned} \Delta V_s(k) &= V_s(k+1) - V_s(k) \\ &= \sum_{i=1}^{N}(s_i(k+1) + s_i(k))(s_i(k+1) - s_i(k)). \end{aligned} \tag{21}$$

The condition to stabilize the closed-loop system is $\Delta V_s(k) < 0$ when $s_i(k) \neq 0$. The equivalent stability conditions are

$$(s_i(k+1) - s_i(k))\,\mathrm{sgn}(s_i(k)) < 0, \tag{22}$$

$$(s_i(k+1) + s_i(k))\,\mathrm{sgn}(s_i(k)) > 0. \tag{23}$$

Then let us discuss the two cases $s_i(k) > 0$ and $s_i(k) < 0$.

Case 1: $s_i(k) > 0$. From (19), one gets

$$\begin{aligned} s_i(k+1) - s_i(k) = &- a_{i0}KB(u_0(k) - u_{min}) \\ &- \varepsilon T - qTs_i(k), \end{aligned} \tag{24}$$

$$\begin{aligned} s_i(k+1) + s_i(k) = &- a_{i0}KB(u_0(k) - u_{min}) \\ &+ (2-qT)s_i(k) - \varepsilon T. \end{aligned} \tag{25}$$

For (24), $s_i(k+1) - s_i(k) < 0$ is satisfied. For (25), if $s_i(k+1) + s_i(k) > 0$, it results in

$$s_i(k) > \frac{\varepsilon T + a_{i0}KB(u_0(k) - u_{min})}{2 - qT}. \tag{26}$$

Case 2: $s_i(k) < 0$. From (19), one has

$$\begin{aligned} s_i(k+1) - s_i(k) = &- a_{i0}KB(u_0(k) - u_{max}) \\ &+ \varepsilon T - qTs_i(k), \end{aligned} \tag{27}$$

$$\begin{aligned} s_i(k+1) + s_i(k) = &- a_{i0}KB(u_0(k) - u_{max}) \\ &+ (2-qT)s_i(k) + \varepsilon T. \end{aligned} \tag{28}$$

For (27), the condition $s_i(k+1) - s_i(k) > 0$ is fulfilled. For (28), if $s_i(k+1) + s_i(k) < 0$, it yields

$$s_i(k) < \frac{-\varepsilon T + a_{i0}KB(u_0(k) - u_{max})}{2 - qT} \tag{29}$$

The definitions of u_{min}, u_{max}, and Δ give

$$\frac{\varepsilon T + a_{i0}KB(u_0(k) - u_{min})}{2 - qT} \le \Delta, \tag{30}$$

and

$$\frac{\varepsilon T + a_{i0}KB(u_0(k) - u_{max})}{2 - qT} \ge -\Delta. \tag{31}$$

From the above analysis of the two cases, one can see that when $s_i(k) \ge \Delta$ or $s_i(k) \le -\Delta$, the conditions (22) and (23) are satisfied simultaneously, which means that $s_i(k)$ converges to zero. When $-\Delta < s_i(k) < \Delta$, only (22) is fulfilled. It means that the phenomenon of increasing amplitude chatter around the switching plane will be occurred, but according to conditions (22) and (23), it will be limited by $\pm\Delta$. It can be concluded that the system is bounded stable, namely,

$$\lim_{k\to\infty}|s_i(k)| \le \Delta. \tag{32}$$

Meanwhile, the quasi-sliding mode band (14) can be obtained. The proof is completed.

□

Remark 1. *From* (14)*, it can be concluded that the width of quasi-sliding mode band* 2Δ *decreases with the decreasing of the sampling period* T*. Moreover, one can see that when the leader's control input* $u_0(k)$ *is exactly known, namely,* $u_{max} = u_{min} = u_0(k)$*, the quasi-sliding mode band* Δ *will degrade to* $\Delta' = \varepsilon T/(2 - qT)$*.*

V. Experimental results

A. Simulation platform

In order to make the simulation as close as possible to the reality, ROS and Gazebo are used to establish the multiple UAVs formation tracking simulation platform. As an open source simulator with physic engine, Gazebo supplies a robot simulation environment, where both the gravity, friction, and contact forces are considered. By integrating with ROS, Gazebo simulator has been widely applied in the field of robotics research.

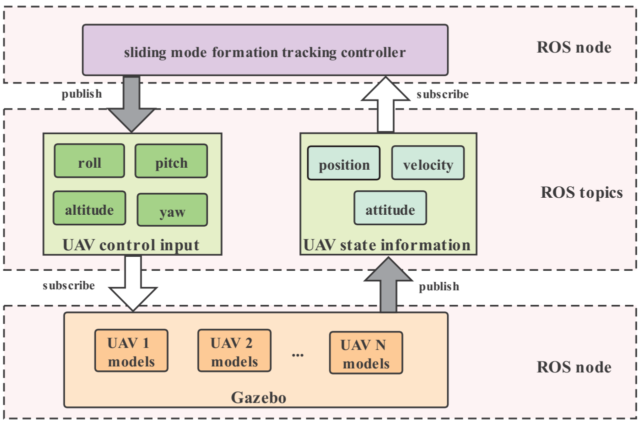

Figure 2: Structure of the formation tracking platform.

Fig. 2 indicates the organization of the multiple UAVs formation tracking platform. Gazebo sends states information such as position, velocity, and attitude of each UAV through ROS topic. The sliding mode formation tracking controller node subscribes to the states, calculates the control inputs of the four channels of pitch, roll, yaw, and altitude of each UAV, and publishes them to the corresponding ROS topic. Gazebo simulates the movement of UAVs according to their control inputs, and displays the trajectory of UAVs in real time.

B. Simulation results

A multi-UAV system with one leader and six followers are taken into account. In order to clarify the movement of the UAVs in the X-Y plane, the positions and velocities in the two directions are all considered. In this case, $n = 2$, the state vector $x_i(k)$, formation vector $f_i(k)$, and control input $u_i(k)$ of UAV i can be rewritten as $x_i(k) = [p_{iX}(k), v_{iX}(k), p_{iY}(k), v_{iY}(k)]^{\mathrm{T}}$, $f_i(k) = [f_{ipX}(k), f_{ivX}(k), f_{ipY}(k), f_{ivY}(k)]^{\mathrm{T}}$, and $u_i(k) = [u_{iX}(k), u_{iY}(k)]^{\mathrm{T}}$, respectively. The 0-1 interaction topology between the UAVs is shown in Fig. 3. One can verify that there exists a directed spanning tree from the leader. Then one has that $H = \mathrm{diag}(1, 0, 0, 0, 0, 0)$, and

$$L = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

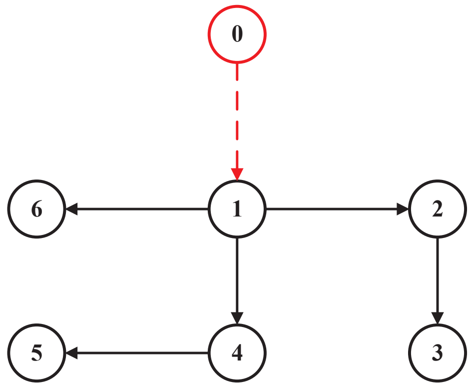

Figure 3: Interaction topology among the UAVs.

The leader makes a circular motion around the origin of coordinates, where $w_l = 0.157$ rad/s and $r_l = 20$ m. Then, one can choose $u_{min} = -1$ and $u_{max} = 1$. The time-varying formation for the followers is a circular motion with radius $r_f = 10$ m, angular velocity $\omega_f = 0.314$ rad/s, and the phase difference $\pi/3$. The formation vector is

$$f_i(k) = \begin{bmatrix} r_f \cos(\omega_f k + \frac{(i-1)\pi}{3}) \\ -\omega_f r_f \sin(\omega_f k + \frac{(i-1)\pi}{3}) \\ r_f \sin(\omega_f k + \frac{(i-1)\pi}{3}) \\ \omega_f r_f \cos(\omega_f k + \frac{(i-1)\pi}{3}) \end{bmatrix}, \ i = 1, 2, \ldots, 6.$$

According to Theorem 1, the parameters of control inputs in (13) for followers are chosen as $\varepsilon = 0.05$, $q = 10$, and $K = [3, 1]$.

The trajectories of seven UAVs in X-Y plane within 40s are shown in Fig. 4, where the positions of UAVs at $t = 0$s and $t = 40$s are represented by round and hexagon markers, respectively. The positions of seven UAVs in horizontal plane at $t = 40$s are indicated in Fig. 5.

According to the red circle in Fig. 5, the six followers reach on the circle with a radius 10 m at $t = 40$ s, and the leader is at the center of the circle. One can see that the desired circular formation motion among the followers is achieved. Therefore, it can be concluded that the desired time-varying formation tracking $f_i(k)$ of the multi-UAV system is realized.

VI. Conclusions

Discrete-time formation tracking problem for multi-UAV system was investigated, where the leader was subject to time-varying control input. Using sliding mode control method, a discrete-time formation tracking controller was constructed. Sufficient conditions for multi-UAV system to complete the

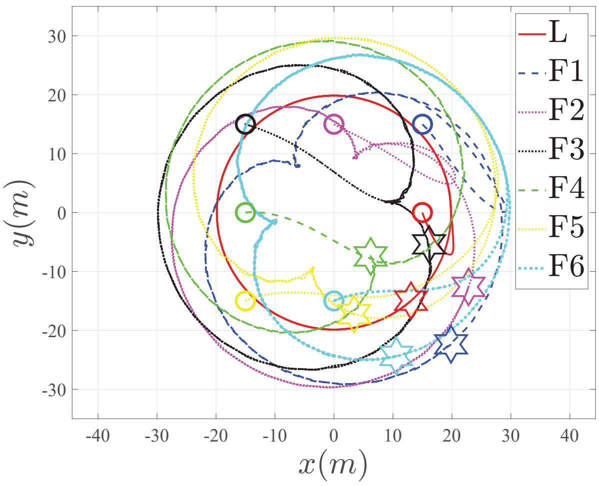

Figure 4: Trajectories of seven UAVs within 40 s.

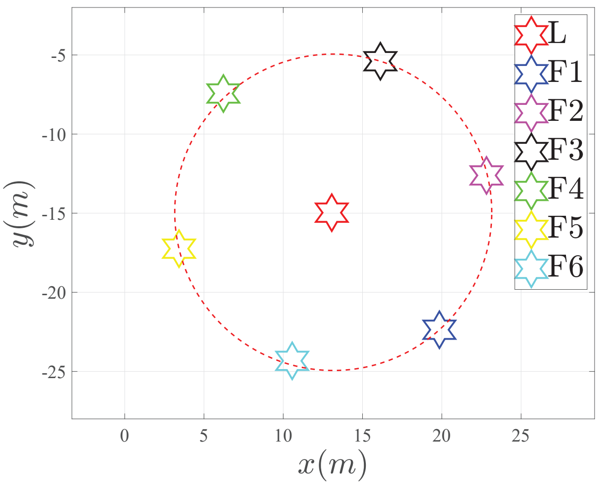

Figure 5: Positions of seven UAVs at 40 s.

time-varying formation tracking control were given, where the quasi sliding mode band was derived. The results of formation tracking experiment with seven quadrotor UAVs in Gazebo verified the effectiveness of the designed protocol.

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