Formation tracking control for second-order nonlinear multi-agent system with unknown maneuvering leader

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***Abstract*—Time-varying formation tracking control problems for multi-agent systems with second-order nonlinear dynamics are investigated in this paper, where a leader with unknown dynamic control input is considered. By utilizing sliding mode control approach, a distributed formation tracking controller on the basis of the neighboring state information is designed. Based on Lyapunov stability theory, sufficient conditions for the realization of predefined formation tracking are summarized. A numerical simulation with six agents is performed to demonstrate the effectiveness of the proposed formation tracking protocol.**

***Index Terms*—formation tracking, multi-agent system, second-order nonlinear dynamics, dynamic leader**

I. INTRODUCTION

Over the past few years, formation control problems of multi-agent systems have been extensively studied. With the collaboration between several agents, the multi-agent system has the ability to perform more complicated missions in more complex environments than individual agents. Unmanned Aerial Vehicles (UAVs) and Unmanned Ground Vehicles (UGVs), and Autonomous Underwater Vehicles (AUVs) are three typical agents. As an important branch of cooperative control technologies, formation control technologies can be applied to various domains, such as target enclosing [1] and source seeking [2].

Formation control problems of first-order [3], second-order [4], [5], and high-order [6]–[8] multi-agent system have been investigated. While in the above studies, just the formation control problems are taken into account. In practical applications, the multiple agents need not only to realize a predefined formation, but also to track a trajectory generated by a virtual or real leader. In this case, the formation tracking problems arise. In [9], the multi-agent system with double integrator dynamics formation tracking problem is studied, where the sufficient conditions for the stabilization of closed-loop system with designed formation tracking controller are given. A sliding mode control strategy is developed in [10] to guarantee that the multi-agent system with double integrator can complete the expected formation tracking in finite time. Sufficient conditions for multi-agent system with double integrator under directed interaction topology to realize the formation tracking are given in [11].

However, the researches in [9]–[11] are based on linear models of multi-agent system, such as with double integrators or multiple integrators dynamics. While in real physical system, the non-linearity is universal. So it is more significant to focus on and deal with the formation tracking problems of multi-agent system with nonlinear dynamics. Formation tracking problems for second-order nonlinear multi-agent system are investigated in [12], where neural network is utilized in the proposed formation protocol design. A distributed observer-based control strategy is employed in [13] to handle the formation problems of nonlinear multi-agent system with double integrators dynamics, where the time delays among the agents are taken into account. Formation tracking problems of nonlinear multi-agent system are studied in [14], where an adaptive iterative learning approach is proposed. A state observer based protocol is designed in [15] to ensure the realization of multi-agent system formation tracking with general nonlinear dynamics.

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It should be noted that in the above studies of formation tracking problems, it is assumed that the leader agent is without any control input, or with known control input. But in actual applications, the multi-agent system need to track not only the cooperative target, but also the noncooperative target. The above assumption is not valid when the leader agent is noncooperative one. As far as we know, the research of the time-varying formation tracking control problems of nonlinear second-order multi-agent system with unknown dynamic leader is still open.

Time-varying formation tracking control problems for second-order nonlinear multi-agent system with a dynamic leader are studied in this paper. Based on the neighboring state information, a formation tracking controller using the sliding mode control approach is given. The convergence of the closed-loop system with the designed controller is demonstrated with the help of Lyapunov stability theory. To verify the validity of the proposed formation tracking method, a numerical simulation with six agents is conducted.

Compared with the previous researches on formation tracking control problems of multi-agent system, the main contributions of this paper are threefold. First, the formation tracking vectors are time-varying. Compared with the static formation problems studied in [10], [12], [13], time-varying formation tracking technologies have the potential for broader application. Second, the nonlinear dynamics of multi-agent system is considered. The design and analyses of formation tracking protocol are more complicated, and the theoretical results are more practical. while only linear models are considered in [9]–[11]. Third, a leader with maneuvering acceleration is considered in the formation tracking control problems of multi-agent system, where only the bounds of maneuvering acceleration need to be known. While in [12]–[14], it is assumed that the leader is without any control input or with known control input, which is difficult to be satisfied in practical engineering.

Notations: sgn represents the symbolic function; $\operatorname{sgn}(A) = [\operatorname{sgn}(A_1), \operatorname{sgn}(A_2), \dots, \operatorname{sgn}(A_n)]^{\mathrm{T}}$, if $A = [A_1, A_2, \dots, A_n] \in \mathbb{R}^n$. In addition, for a vector or matrix A, $|A|$ and $\|A\|$ denote the 1-norm and 2-norm, respectively. $\mathbf{1}_N$ denotes a N-dimensional column vector with all elements being 1. The Kronecker product is indicated by $\otimes$.

II. Preliminaries and System Modeling

A. Graph theory

Consider a multi-agent system with one leader labeled 0 and N followers labeled $1, 2, \dots, N$. The interaction communication topology among the N followers can be described by a weighted directed graph $\mathcal{G} = (\mathcal{W}, \mathcal{E}, \mathcal{A})$, where $\mathcal{W} = \{w_1, w_2, \dots, w_N\}$ denotes the set of nodes, $\mathcal{E} = \{e_{ij} = (w_j, w_i), w_i, w_j \in \mathcal{W}\}$ represents the set of edges, and $\mathcal{A} = [a_{ij}]_{N\times N}$ is the weighted adjacency matrix with $i, j \in \{1, 2, \dots, N\}$. In addition, e_{ij} denotes the edge formed by nodes w_j and w_i, where w_j and w_i are called the parent node and child node, respectively. Moreover, $a_{ij} > 0$ represents the weight of edge e_{ij} if $e_{ij} \in \mathcal{E}$, and $a_{ij} = 0$ if not. Besides, one assumes that $a_{ii} = 0$, $\forall i = 1, 2, \dots, N$. A *directed path* between nodes w_i and w_j is defined by a series of edges $(w_i, w_{i1}), (w_{i1}, w_{i2}), \dots, (w_{il}, w_j)$, where w_{ik} $(k = 1, 2, \dots, l)$ are different nodes of the graph. A graph $\mathcal{G}$ is said to have a *directed spanning tree* if there exists one node which has directed paths to the other nodes. The in-degree of node w_i is defined as $\deg_{\text{in}}(w_i) = \sum_{j=1,j\neq i}^{N} a_{ij}$. Then the in-degree matrix $\mathcal{D}$ and the Laplacian matrix L are defined as $\mathcal{D} = \operatorname{diag}(\deg_{\text{in}}(w_i), i = 1, 2, \dots, N)$ and $L = \mathcal{D} - \mathcal{A}$, respectively.

Assume that the communication between the leader and followers is unidirectional, which means that the followers can get the status of the leader, but otherwise it is not. The interaction weight between the leader and follower i is denoted by a_{i0}. $a_{i0} > 0$ if the follower i can get the status of the leader, and $a_{i0} = 0$ if not. In addition, denote $H = \operatorname{diag}(a_{10}, a_{20}, \dots, a_{N0})$ and denote $L_H = L + H$.

Lemma 1. *If the graph $\mathcal{G}$ contains a directed spanning tree from the leader, then the matrix L_H is invertible.*

Lemma 2. *Consider a nonlinear system $\dot{y} = g(y)$, $g(0) = 0$, and a positive definite function $V(y) \in \mathbb{R}$. If there exist two constants c and γ such that*

$$\dot{V}(y) + cV^{\gamma}(y) \leq 0, \tag{1}$$

with $c > 0$ and $0 < \gamma < 1$. Then $V(y)$ can reach zero in a finite period, where the finite setting time T depending on the initial state is

$$T \leq \frac{V^{(1-\gamma)}(y(0))}{c(1-\gamma)}. \tag{2}$$

Lemma 3. *For a continuous positive function $V(t) \in \mathbb{R}$, namely, $V(t) \geq 0$ with $\forall t > 0$, if there exist two positive constants σ and β such that*

$$\dot{V}(t) \leq -\beta V(t) + \sigma, \tag{3}$$

then $V(t)$ is upper bounded and satisfies

$$V(t) \leq V(0)e^{-\beta t} + \frac{\sigma}{\beta}(1 - e^{-\beta t}). \tag{4}$$

B. System description

The nonlinear dynamics of the leader ($i = 0$) and followers ($i = 1, 2, \dots, N$) can be described as

$$\begin{aligned} \dot{x}_i(t) &= v_i(t), \\ \dot{v}_i(t) &= f(x_i(t), v_i(t), t) + u_i(t), \end{aligned} \tag{5}$$

where $x_i(t) \in \mathbb{R}^m$, $v_i(t) \in \mathbb{R}^m$, and $u_i(t) \in \mathbb{R}^m$ are the position, velocity, and control input for agent i at time t, respectively. The term $f(x_i(t), v_i(t), t)$ is the corresponding intrinsic nonlinear dynamics.

Assumption 1. *For the leader's dynamic input $u_0(t)$, there exists a constant $\hat{u}_0 > 0$ such that $\forall t \geq 0, |u_0(t)| \leq \hat{u}_0$.*

Assumption 2. *There exist two positive constants* ρ_1 *and* ρ_2 *such that for* $\forall i = 1, 2, \ldots, N$*, one has*

$$\begin{aligned}&\|f(x_i(t), v_i(t), t) - f(x_0(t), v_0(t), t)\| \\ &\quad \le \rho_1\|x_i(t) - x_0(t)\| + \rho_2\|v_i(t) - v_0(t)\|.\end{aligned} \tag{6}$$

Assumption 3. *The directed graph* $\mathcal{G}$ *contains at least a spanning tree from the leader agent.*

III. Problem statement and transformation

Definition 1. *For system* (5)*, the desired formation tracking* $\{h_{xi}(t), h_{vi}(t)\}$ *for each agent* i *(*$i = 1, 2, \ldots, N$*) is realized, if the following two conditions are met simultaneously*

$$\begin{aligned}&\lim_{t\to\infty}(x_i(t) - h_{xi}(t) - x_0(t)) = \mathbf{0}, \\ &\lim_{t\to\infty}(v_i(t) - h_{vi}(t) - v_0(t)) = \mathbf{0},\end{aligned} \tag{7}$$

where $h_{xi}(t) \in \mathbb{R}^m$ *and* $h_{vi}(t) \in \mathbb{R}^m$ *are the corresponding time-varying position and velocity formation components, respectively. In addition, the equation* $\dot{h}_{xi}(t) = h_{vi}(t)$ *is required for the desired formation vectors.*

For follower i ($i = 1, 2, \ldots, N$), the position and velocity formation tracking errors are defined as

$$\begin{aligned}e_{xi}(t) = &\sum_{j=1, j\neq i}^{N} a_{ij}(x_i(t) - h_{xi}(t) - x_j(t) + h_{xj}(t)) \\ &+ a_{i0}(x_i(t) - h_{xi}(t) - x_0(t)), \\ e_{vi}(t) = &\sum_{j=1, j\neq i}^{N} a_{ij}(v_i(t) - h_{vi}(t) - v_j(t) + h_{vj}(t)) \\ &+ a_{i0}(v_i(t) - h_{vi}(t) - v_0(t)).\end{aligned} \tag{8}$$

Let $x(t) = [x_1(t)^T, \ldots, x_N(t)^T]^T$, $v(t) = [v_1(t)^T, \ldots, v_N(t)^T]^T$, $h_x(t) = [h_{x1}(t)^T, \ldots, h_{xN}(t)^T]^T$, $h_v(t) = [h_{v1}(t)^T, \ldots, h_{vN}(t)^T]^T$, $u(t) = [u_1(t)^T, \ldots, u_N(t)^T]^T$, $\epsilon_x(t) = [e_{x1}(t)^T, \ldots, e_{xN}(t)^T]^T$, $\epsilon_v(t) = [e_{v1}(t)^T, \ldots, e_{vN}(t)^T]^T$, $F = [f(x_1, v_1, t)^T, \ldots, f(x_N, v_N, t)^T]^T$, and $f_0 = f(x_0, v_0, t)$. Then, denote

$$\begin{aligned}\tilde{x}(t) &= x(t) - h_x(t) - \mathbf{1}_N \otimes x_0(t), \\ \tilde{v}(t) &= v(t) - h_v(t) - \mathbf{1}_N \otimes v_0(t).\end{aligned} \tag{9}$$

Using the notations of $\epsilon_x(t)$, $\epsilon_v(t)$, $\tilde{x}(t)$, and $\tilde{v}(t)$, (8) can be written as

$$\begin{aligned}\epsilon_x(t) &= L_H \otimes I_m \cdot \tilde{x}(t), \\ \epsilon_v(t) &= L_H \otimes I_m \cdot \tilde{v}(t).\end{aligned} \tag{10}$$

Taking the derivative of (10), one gets the following formation tracking error system

$$\begin{aligned}\dot{\epsilon}_x(t) &= \epsilon_v(t), \\ \dot{\epsilon}_v(t) &= L_H \otimes I_m \cdot (F + u(t) - \dot{h}_v(t) \\ &\quad - \mathbf{1}_N \otimes (f_0 + u_0(t))).\end{aligned} \tag{11}$$

Lemma 4. *For multi-agent system* (5)*, the desired time-varying formation tracking will be realized if the error system* (11) *converge to zero.*

Proof. When the error system (11) converges to zero, it yields $\epsilon_x \to \mathbf{0}$ and $\epsilon_v \to \mathbf{0}$ as $t \to \infty$. According to Lemma 1, one has

$$\begin{aligned}&\lim_{t\to\infty}\tilde{x}(t) = \mathbf{0}, \\ &\lim_{t\to\infty}\tilde{v}(t) = \mathbf{0}.\end{aligned} \tag{12}$$

From the expression of $\tilde{x}(t)$ and $\tilde{v}(t)$, one has

$$\begin{aligned}&\lim_{t\to\infty} x(t) - h_x(t) - \mathbf{1}_N \otimes x_0(t) = \mathbf{0}, \\ &\lim_{t\to\infty} v(t) - h_v(t) - \mathbf{1}_N \otimes v_0(t) = \mathbf{0}.\end{aligned} \tag{13}$$

According to Definition 1, it can be concluded that the desired formation tracking $\{h_x(t), h_v(t)\}$ is realized. The proof is completed. □

The control objective of this paper is to design an appropriate protocol to ensure that the desired formation tracking can be achieved.

IV. Formation tracking protocol design and analyses

The distributed formation tracking controller for agent i ($i = 1, 2, \ldots, N$) is designed as

$$u_i(t) = u_{ai}(t) + u_{bi}(t) + u_{ci}(t), \tag{14}$$

where

$$u_{ai}(t) = \dot{h}_{vi}(t), \tag{15}$$

$$u_{bi}(t) = (a_{i0} + \sum_{j=1, j\neq i}^{N} a_{ij})^{-1}\Big[\sum_{j=1, j\neq i}^{N} a_{ij}u_{bj}(t) - \mu e_{vi}(t)\Big], \tag{16}$$

$$\begin{aligned}u_{ci}(t) = &(a_{i0} + \sum_{j=1, j\neq i}^{N} a_{ij})^{-1}\Big[\sum_{j=1, j\neq i}^{N} a_{ij}u_{cj}(t) \\ &- (g + c_0 + c_1)\,\mathrm{sgn}(e_{vi}(t) + \mu e_{xi}(t))\Big],\end{aligned} \tag{17}$$

with μ, g, c_0, and c_1 four control parameters.

Theorem 1. *The multi-agent system* (5) *can realize the desired time-varying formation tracking under the protocol* (14) *if the parameters* g *and* c_0 *satisfy*

1) $g = \omega(\rho_1\|\epsilon_x\| + \rho_2\|\epsilon_v\|)$*, with* $\omega = \|L_H\|\|L_H^{-1}\|$*,*
2) $c_0 = \|\boldsymbol{h} \otimes \hat{u}_0\|$*, with* $\boldsymbol{h} = [a_{10}, a_{20}, \ldots, a_{N0}]^T$*,*
3) μ *and* c_1 *are two arbitrary positive constants.*

Proof. Define the sliding mode fucntion S as

$$S(t) = \epsilon_v(t) + \mu\epsilon_x(t). \tag{18}$$

Then reorganize the protocol (14) as a more compact form

$$u(t) = u_a(t) + u_b(t) + u_c(t), \tag{19}$$

where

$$u_a(t) = \dot{h}_v(t), \tag{20}$$

$$u_b(t) = -\mu L_H^{-1} \otimes I_m \cdot \epsilon_v(t), \tag{21}$$

$$u_c(t) = -L_H^{-1} \otimes I_m \cdot \{\mathrm{sgn}(S(t)) \cdot (g + c_0 + c_1)\}. \tag{22}$$

The Lyapunov candidate function for $S(t)$ is chosen as

$$V_s(t) = \frac{1}{2}S(t)^T S(t). \tag{23}$$

Taking the derivative of (18), one has

$$\begin{aligned}\dot{S}(t) &= \mu\dot{\epsilon_x}(t) + \dot{\epsilon_v}(t) \\ &= \mu\epsilon_v(t) + L_H \otimes I_m \cdot (F + u(t) - \dot{h}_v(t) \\ &\quad - \mathbf{1}_N \otimes (f_0 + u_0(t))).\end{aligned} \tag{24}$$

Then the derivative of $V_s(t)$ is

$$\begin{aligned}\dot{V}_s(t) &= S(t)^T \dot{S}(t) \\ &= S(t)^T\{\mu\epsilon_v(t) + L_H \otimes I_m \cdot (F + u(t) \\ &\quad - \dot{h}_v(t) - \mathbf{1}_N \otimes (f_0 + u_0(t)))\}.\end{aligned} \tag{25}$$

Substituting (19) into (25) yields

$$\begin{aligned}\dot{V}_s(t) = S(t)^T\{&L_H \otimes I_m \cdot (F + u_c(t) \\ &- \mathbf{1}_N \otimes (f_0 + u_0(t)))\}.\end{aligned} \tag{26}$$

Using the facts on the Kroneckor product, one has

$$\begin{aligned}\|L_H \otimes I_m\| &= \sqrt{\lambda_{max}(L_H \otimes I_m)^T(L_H \otimes I_m)} \\ &= \sqrt{\lambda_{max}(L_H^T L_H) \otimes I_m} \\ &= \sqrt{\lambda_{max}(L_H^T L_H)} \\ &= \|L_H\|.\end{aligned} \tag{27}$$

For the nonlinear function, the following inequality can be obtained

$$\begin{aligned}\|F - \mathbf{1}_N \otimes f_0\| &= \|[(f(x_1,v_1,t) - f(x_0,v_0,t))^T, \\ &\qquad \ldots, (f(x_N,v_N,t) - f(x_0,v_0,t))^T]^T\| \\ &\le \|[\|f(x_1,v_1,t) - f(x_0,v_0,t)\|, \\ &\qquad \ldots, \|f(x_N,v_N,t) - f(x_0,v_0,t)\|]^T\| \\ &\le \|[\rho_1\|x_1 - x_0\| + \rho_2\|v_1 - v_0\|, \\ &\qquad \ldots, \rho_1\|x_N - x_0\| + \rho_2\|v_N - v_0\|]^T\| \\ &\le \rho_1\|[\|x_1 - x_0\|, \ldots, \|x_N - x_0\|]^T\| \\ &\qquad + \rho_2\|[\|v_1 - v_0\|, \ldots, \|v_N - v_0\|]^T\| \\ &\le \rho_1\|\tilde{x}\| + \rho_2\|\tilde{v}\|.\end{aligned} \tag{28}$$

Thus, from (27) and (28), one has

$$\begin{aligned}&\|L_H \otimes I_m \cdot (F - \mathbf{1}_N \otimes f_0)\| \\ &\le \|L_H \otimes I_m\| \cdot \|F - \mathbf{1}_N \otimes f_0\| \\ &\le \|L_H\| \cdot (\rho_1\|\tilde{x}\| + \rho_2\|\tilde{v}\|) \\ &\le \|L_H\|\|L_H^{-1}\| \cdot (\rho_1\|\epsilon_x\| + \rho_2\|\epsilon_v\|) \\ &\le \omega(\rho_1\|\epsilon_x\| + \rho_2\|\epsilon_v\|).\end{aligned} \tag{29}$$

Then it yields the following inequality for $\dot{V}_s(t)$

$$\begin{aligned}\dot{V}_s(t) &= S^T\{L_H \otimes I_m \cdot (F + u_c - \mathbf{1}_N \otimes (f_0 + u_0))\} \\ &\le \|S\| \cdot \|L_H \otimes I_m \cdot (F - \mathbf{1}_N \otimes f_0)\| \\ &\qquad + S^T\{L_H \otimes I_m \cdot (u_c - \mathbf{1}_N \otimes u_0)\} \\ &\le \omega\|S\|(\rho_1\|\epsilon_x\| + \rho_2\|\epsilon_v\|) \\ &\qquad + S^T\{\boldsymbol{h} \otimes (-u_0) - \mathrm{sgn}(S) \cdot (g + c_0 + c_1)\} \\ &\le \omega\|S\|(\rho_1\|\epsilon_x\| + \rho_2\|\epsilon_v\|) + \|S\| \cdot \|\boldsymbol{h} \otimes \hat{u}_0\| \\ &\qquad - |S| \cdot (g + c_0 + c_1) \\ &\le \omega\|S\|(\rho_1\|\epsilon_x\| + \rho_2\|\epsilon_v\|) + \|S\| \cdot \|\boldsymbol{h} \otimes \hat{u}_0\| \\ &\qquad - \|S\| \cdot (g + c_0 + c_1) \\ &\le \omega\|S\|(\rho_1\|\epsilon_x\| + \rho_2\|\epsilon_v\|) - g\|S\| + \|S\| \cdot \|\boldsymbol{h} \otimes \hat{u}_0\| \\ &\qquad - c_1\|S\| - c_0\|S\| \\ &\le -c_1\|S\| \\ &\le -\frac{\sqrt{2}}{2}c_1 V_s^{\frac{1}{2}}(t).\end{aligned} \tag{30}$$

Since $c_1 > 0$, according to Lemma 2, one can see that the error system (11) can reach the switching plane $S(t) = \mathbf{0}$, and maintain on it.

On the switching plane $S(t) = \mathbf{0}$, the definition of $S(t)$ yields

$$\mu\epsilon_x(t) + \epsilon_v(t) = \mathbf{0}, \tag{31}$$

and then

$$\mu\epsilon_x(t) = -\epsilon_v(t) = -\dot{\epsilon}_x(t). \tag{32}$$

The Lyapunov function for error system (11) is chosen as

$$V_{\epsilon_x}(t) = \frac{1}{2}\epsilon_x(t)^T\epsilon_x(t), \tag{33}$$

Then, one can obtain

$$\dot{V}_{\epsilon_x}(t) = \epsilon_x(t)^T\dot{\epsilon}_x(t) = -\mu\epsilon_x(t)^T\epsilon_x(t) = -2\mu V_{\epsilon_x}(t). \tag{34}$$

From Lemma 3, one can get $\epsilon_x(t) \to 0$ as $t \to 0$. In addition, since on the switching plane $S(t) = \mathbf{0}$, one has $\epsilon_v(t) = -\mu\epsilon_x(t)$. Thus, it yields $\epsilon_v(t) \to 0$ as $t \to 0$. Then, by Lemma 4, it can be obtained that under the protocol (19), the desired time-varying formation tracking is realized as the time tends to infinity. The proof is completed. □

V. Numerical simulation

A. simulation settings

A multi-agent system with one leader and five followers is taken into account. In order to clarify the movement of the agents in the X-Y plane, the positions and velocities in the two directions are all considered. In this case, $m = 2$, the position $x_i(t)$, velocity $v_i(t)$, position formation component $h_{xi}(t)$, velocity formation component $h_{vi}(t)$, and control input $u_i(t)$ of agent i can be written as $x_i(t) = [x_{iX}(t), x_{iY}(t)]^T$, $v_i(t) = [v_{iX}(t), v_{iY}(t)]^T$, $h_{xi}(t) = [h_{xiX}(t), h_{xiY}(t)]^T$, $h_{vi}(t) = [h_{viX}(t), h_{viY}(t)]^T$, $u_i(t) = [u_{iX}(t), u_{iY}(t)]^T$, respectively. The nonlinear function is $f(x_i, v_i, t) = -x_i\cos(t) - v_i\sin(t) - \cos(v_i)$, then one has $\rho_1 = 2$ and $\rho_2 = 2$.

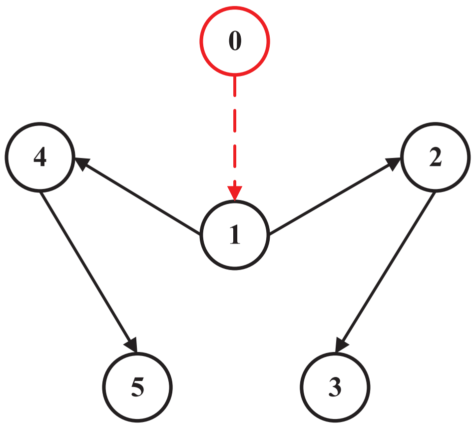

Figure 1: Interaction topology among the six agents.

The 0-1 topology among the six agents is displayed in Fig. 1. One can verify that there exists a directed spanning tree from the leader. Then one has $H = \mathrm{diag}(1, 0, 0, 0, 0)$, and

$$L = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}.$$

The leader makes a circular motion around the origin of the coordinates, where $w_l = 0.314$ rad/s and $r_l = 20$ m. Then, one can choose $\hat{u}_0 = 2$. The predefined formation shape for the followers is a circular motion with radius $r_f = 10$ m and angular velocity $\omega_f = 0.628$ rad/s. The corresponding position formation $h_{xi}(t)$ and velocity formation $h_{vi}(t)$ vectors for follower i $(i = 1, 2, \ldots, N)$ are $h_{xi}(t) = \begin{bmatrix} r_f \cos(\omega_f k + \frac{2(i-1)\pi}{5}) \\ r_f \sin(\omega_f k + \frac{2(i-1)\pi}{5}) \end{bmatrix}$ and $h_{vi}(t) = \begin{bmatrix} -\omega_f r_f \sin(\omega_f k + \frac{2(i-1)\pi}{5}) \\ \omega_f r_f \cos(\omega_f k + \frac{2(i-1)\pi}{5}) \end{bmatrix}$, respectively. One can see that the condition $\dot{h}_{xi}(t) = h_{vi}(t)$ is satisfied.

From Theorem 1, the control parameters $\mu = 0.3$ and $c_1 = 0.5$ are chosen for protocol (14).

B. *simulation results*

Fig. 2 shows the trajectories of agents in horizontal plane within 40 s, where the locations of agents at $t = 0$ s and $t = 40$ s are represented by round and hexagon markers, respectively. The detailed positions of six agents in horizontal plane at $t = 40$ s are indicated in Fig. 3. The switching function $s_1(t)$ of follower 1 is represented in Fig. 4. The position and velocity formation tracking errors of follower 1 in X direction are indicated in Fig. 5. Taking the follower 1 as an example, the switching functions and formation tracking errors of the other followers are similar to those of the follower 1.

According to the red circle in Fig. 3, the five followers reach on the circle with a radius 10 m at $t = 40$ s, and the leader is at the center of the circle. One can get that the predefined circular formation motion among the followers is achieved. In addition, from Fig. 4 and Fig. 5, it can be obtained that the switching function and formation tracking error function decrease rapidly at the beginning and eventually converge to a bounded region. Therefore, it can be concluded that the expected formation tracking is realized.

VI. Conclusions

Formation tracking control problems for nonlinear second-order multi-agent system were investigated, where the leader was subject to unknown dynamically changing acceleration. By utilizing the neighboring state information, a distributed formation tracking controller based on sliding mode control method was constructed. Sufficient conditions for multi-agent system to complete the desired time-varying formation tracking were given. The numerical simulation with six agents verified the usefulness of the designed formation tracking approach.

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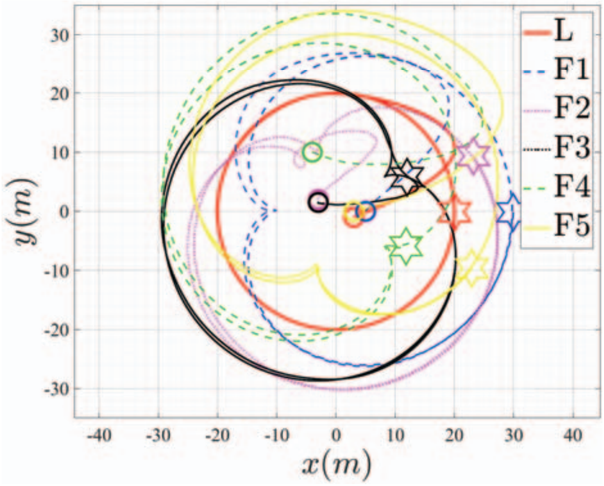

Figure 2: Trajectories in X-Y plane of six agents within 40 s.

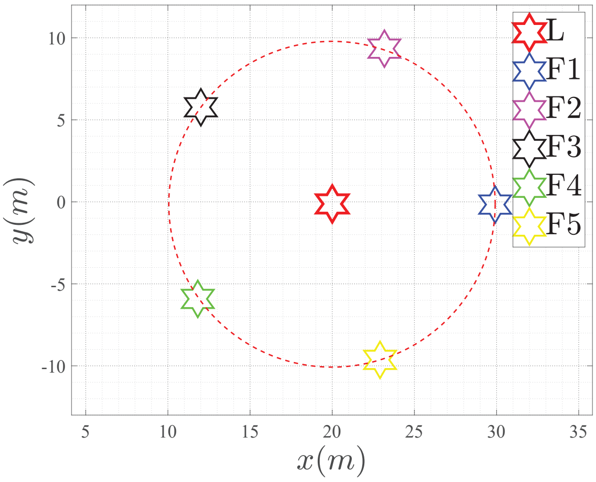

Figure 3: Positions in X-Y plane of six agents at 40 s.

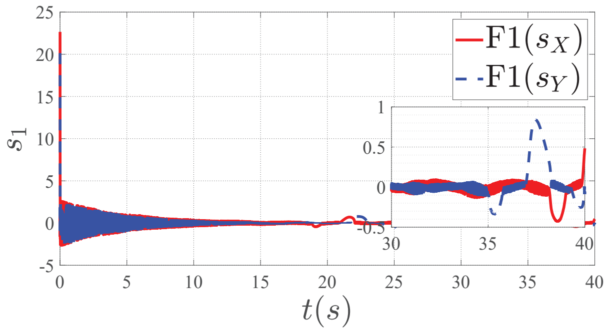

Figure 4: Switching function of agent 1 within 40 s.

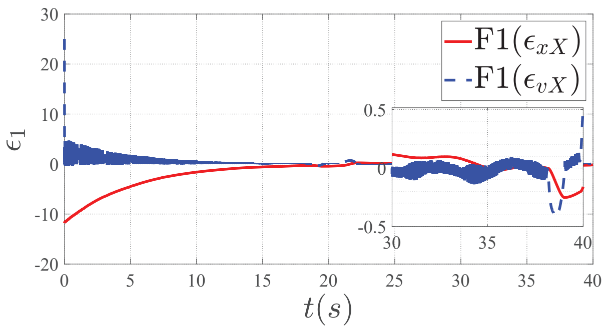

Figure 5: Error function of agent 1 within 40 s.

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