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Bipartite Antagonistic Time-Varying Formation Tracking for Multi-agent System

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Abstract: Bipartite antagonistic time-varying formation tracking problems for multi-agent systems with one dynamic leader is investigated in this paper, where the followers track the leader while achieving a predifined time-varying formation, and one group of followers tracks the leader while another group of followers tracks the leader at the opposite position to the origin. The competitive interactions among the followers and the external input of the leader are both taken into consideration. A distributed bipartite antagonistic time-varying formation tracking protocol is constructed by using the neighboring state information. Sufficient conditions for multi-agent system to achieve the bipartite formation tracking are given. An approach to extend the feasible set of formation is proposed and an algorithm to design the protocol is introduced. A numerical simulation is provided to demonstrate the effectiveness of the protocol.

Key Words: Bipartite graph, Antagonistic network, Time-varying formation tracking, Multi-agent system

1 Introduction

During the past few decades, cooperative control problems of multi-agent systems have been extensively studied and applied in various fields, for example, cooperative localization [1], target enclosing [2], and collaboration of sensor network [3]. Formation control, which requires that the agents of the system achieve a predefined formation, is one of the most attractive categories of the cooperative control due to its application potential in the field of unmanned aerial vehicle (UAV) [4] and spacecraft group control [5] .

Time-invariant formation problems for second-order multi-agent systems with undirected graph are presented and sufficient conditions to realize a formation are given in [6]. Distributed time-varying formation tracking protocol for linear second-order multi-agent systems is proposed in [7]. A formation control approach for linear second-order multi-agent systems with time-varying delays are addressed in [8]. Formation control of multi-agent systems with stochastic switching topology and time-varying communication delays is presented in [9]. A time-varying formation control for unmanned aerial vehicles system with switching interaction topologies is addressed in [10].

However, most of the aformentioned research focus on directed graph or an undirected graph. Antagonistic interaction can be apppear in the formation control problem. In [11], necessary and sufficient conditions for the first-order multi-agent systems to realize a bipartite consensus are given. Bipartite flock control problem for multi-agent system is studied and an algorithm to guarantee the bipartite behavior is given in [12]. A bipartite containment tracking problem for multi-agent systems with signed graph is investigated in [13]. Adaptive bipartite tracking problem for double integrator multi-agent system is studied and convergence error is analyzed in [14]. Distributed bipartite tracking consensus problem for linear multi-agent systems with dynamic leader is investigated in [15]. In [16], bipartite consensus problem for multi-agent system with input saturation is studied.

Based on formation control and bipartite antagonistic topology, bipartite antagonistic time-varying formation tracking problem for multi-agent systems with one leader is investigated in this paper. First, antagonistic interaction among followers and external control input for leader are taken into account. Then, a distributed bipartite antagonistic time-varying formation tracking protocol is proposed, where both the desired formation vector and the tracking trajectories can vary over time. Secondly, sufficient conditions for multi-agent systems to realize bipartite antagonistic time-varying formation tracking are proposed. In addition, the feasible set of formations can be extended by introducing a auxiliary constant matrix. Stability of the close-loop system is discussed by using Lyapunov stability theory. Finally, a numerical simulation is given to illustrate the effectivenss of the protocol.

Compared with the previous works, the new contributions of this paper are threefold. First, the tracking trajectory and the formation vector can be time-varying. Second, cooperative and antagonistic interactions among the followers are taken into account and the graph describing the antagnostic topology is more complex. Third, in contrast to bipartite formation approaches in most of the paper, dynamic leader is consid-

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ered in this paper. In practical applications, the tracked target usually has external acceleration.

This paper is organized as follows. Preliminaries of graph theory and the problem transformation are presented in Section 2. Bipartite antagonistic time-varying formation tracking problem analysis, protocol design and proof of stability are given in Section 3. An algorithm to design the protocol is introduced in Section 4. A numerical simulation example is shown for illustration in Section 5. Finally, in Section 6, conclusions are given .

Throughout this paper, for simplicity of notation, $\otimes$ represents the Kronecker product and $\mathbb{R}^{M\times N}$ represents the set of real matrices with M rows and N columns. Notation $\text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_N)$ denotes a diagnal matrix with σ_i as its i-th diagnal element. 1_N denotes a column vector of size N with constant 1 as its element. Symbol $\text{sgn}(A)$ represents the sympolic function, $\text{sgn}(A) = [\text{sgn}(A_1), \text{sgn}(A_2), \ldots, \text{sgn}(A_N)]^T$ if $A = [A_1, A_2, \ldots, A_N]^T \in \mathbb{R}^N$ is a real vector. Notation $||A||$ and $||A||_1$ are used to represent the Euclidean norm and 1-norm of a real vector A, respectively.

2 Preliminaries and problem description

2.1 Basic properties of graph and bigraph theory

A signed directed graph with N nodes is denoted by $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$ is the set of nodes, $\mathcal{E} = \{\mathcal{E}_{ij} = (v_i, v_j),\ \mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}\}$ is the set of edges and $\mathcal{A} = (a_{ij})_{N\times N}$ is a signed weighted adjacency matrix. The signed weight a_{ij} could be positive or negative, $a_{ij} = \mathcal{E}_{ij}$ if $\mathcal{E}_{ij} \in \mathcal{E}$, $a_{ij} = 0$ if not. In addition, assume that $a_{ii} = 0,\ i = 1, 2, \ldots, N$. The set of neighbor node v_i is defined as $\mathcal{N}_i = \{v_i \in \mathcal{V},\ \mathcal{E}_{ij} = (v_i, v_j) \in \mathcal{E}\}$. The graph $\mathcal{G}$ is called undirected if for $\forall\ i, j \in \{1, 2, \ldots, N\}$, there has $a_{ij} = a_{ji}$. There exists a *directed path* between v_i and v_j if there exists a series of edges $(v_i, v_{i_1}), (v_{i_1}, v_{i_2}), \ldots (v_{i_l}, v_j)$ with v_{i_k} $(k = 1, 2, \ldots, l)$ different nodes of the graph $\mathcal{G}$. An undirected graph G is said *connected* if there exists a path for any pair of nodes (v_i, v_j), where $i, j \in \{1, 2, \ldots, N\}$. A directed graph $\mathcal{G}$ has a *directed spanning tree* if there is at least one node which has directed paths to all the other nodes. The in-degree of a node v_i for a signed graph is defined as $\deg_{\text{in}}(v_i) = \sum_{j=1,j\neq i}^{N} |a_{ij}|$. One observes that for a unsigned graph (a_{ij} is nonnegative), the definition of in-degree degrades to $\deg_{\text{in}}(v_i) = \sum_{j=1,j\neq i}^{N} a_{ij}$. The in-degree matrix $\mathcal{W}$ is defined by $\mathcal{W} = \text{diag}(\deg_{\text{in}}(v_1), \deg_{\text{in}}(v_2), \ldots, \deg_{\text{in}}(v_N))$. The Laplacian matrix L of the graph $\mathcal{G}$ is defined by $L = \mathcal{W} - \mathcal{A}$.

Definition 1. A signed graph $\mathcal{G}$ is called structurally balanced if there exists a bipartition of the nodes, $\mathcal{V}_1$ and $\mathcal{V}_2$ and they satisfy the following conditions:

- $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$ and $\mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$
- $a_{ij} \geq 0$ if $v_i, v_j \in \mathcal{V}_p$ $(p \in \{1, 2\})$
- $a_{ij} \leq 0$ if $v_i \in \mathcal{V}_p,\ v_j \in \mathcal{V}_{3-p}$ $(p \in \{1, 2\})$

From the above definition, one can draw the following lemma to determine the structurally balance property of the graph.

Lemma 1. Define a set of diagonal matrix $\mathcal{D} = \{\Pi = \text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_N), \sigma_i = \pm 1\}$, a signed graph $\mathcal{G}$ is structurally balanced if and only if $\exists \Pi \in \mathcal{D}$, such that $\Pi\mathcal{A}\Pi$ has all nonnegative entries. In addition, Π gives a partition of the nodes: $\mathcal{V}_1 = \{i, \sigma_i > 0\}$, $\mathcal{V}_2 = \{i, \sigma_i < 0\}$.

2.2 Definition of bipartite formation tracking

Consider a agent group with one leader and N followers whose states are represented by $x_0(t)$ and $x_i(t), i \in \{1, \ldots, N\}$, respectively. The dynamic equation of each agent is modeled as follows:

$$\dot{x_i}(t) = Ax_i(t) + Bu_i(t) \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$ represents the state of n-order agent i, $u_i(t) \in \mathbb{R}^s$ represents the control input of the agent i. Matrices $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times s}$ are constant matrices.

Assume that the control input of the leader $u_0(t)$ is bounded. There exists a constant $u_{max} > 0$ such that $\forall t > 0, ||u_0(t)|| < u_{max}$.

The interaction topology among the N followers is described by a signed directed graph $\mathcal{G}$, the Laplacian matrix $L \in \mathbb{R}^{N\times N}$ and the adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ with $i, j \in \{1, 2, \ldots, N\}$ of graph $\mathcal{G}$ are defined as before. Further, the interaction topology among the $N+1$ agents (one leader and N followers) is described by a signed directed graph $\tilde{\mathcal{G}}$, the Laplacian matrix and adjacency matrix are $\tilde{L} \in \mathbb{R}^{(N+1)\times(N+1)}$ and $\tilde{\mathcal{A}} = [a_{ij}] \in \mathbb{R}^{(N+1)\times(N+1)}$ where $i, j \in \{0, 1, \ldots, N\}$, respectively.

Assume that the interaction between the leader and follower is positive or zero, i.e. $a_{k0} \geq 0$. Furthermore, one assumes that the graph $\tilde{\mathcal{G}}$ has a directed spanning tree with the leader being the root and the graph $\mathcal{G}$ is connected and structually balanced.

Definition 2. The multi-agent system (1) is said to a realize bipartite tracking consensus if the following conditions are satisfied:

$$\begin{cases} \lim_{t\to\infty} ||x_i(t) - x_0(t)|| = 0, \forall i \in \mathcal{V}_p \\ \lim_{t\to\infty} ||x_i(t) + x_0(t)|| = 0, \forall i \in \mathcal{V}_{3-p},\ p \in \{1, 2\} \end{cases} \tag{2}$$

For a signed structurally balanced graph $\mathcal{G}$, according to Lemma 1 and Definition 2, one can give an equivalent definition for bipartite tracking consensus.

Definition 3. The multi-agent system (1) with a structurally balanced graph $\mathcal{G}$ is said to realize a antagonistic bipartite tracking consensus if the following conditions are satisfied:

$$\lim_{t\to\infty} ||x_i(t) - \sigma_i x_0(t)|| = 0,\ i \in \{1, 2 \ldots, N\} \tag{3}$$

Let $h(t) = [h_1(t)^T, h_2(t)^T, \cdots, h_N(t)^T]^T$ be the desired bipartite time-varying tracking formation vector for the multi-agent system (1), where $h_i(t) \in \mathbb{R}^n$ is the formation vector for agent i.

Definition 4. The multi-agent system (1) is said to realize a bipartite antagonistic time-varying formation tracking $h(t)$ if the states $x_i(t)$ of the N followers satisfy the following conditions:

$$\begin{cases}\lim_{t\to\infty}||x_i(t)-x_0(t)-h_i(t)||=0, \forall i\in\mathcal{V}_p\\ \lim_{t\to\infty}||x_i(t)+x_0(t)-h_i(t)||=0, \forall i\in\mathcal{V}_{3-p},\ p\in\{1,2\}\end{cases} \tag{4}$$

Similar to the above conversion of the bipartite tracking consensus definiton, an equivalent definition for the bipartite antagonistic time-varying formation tracking can be obtained.

Definition 5. The multi-agent system (1) with a structurally balanced graph $\mathcal{G}$ is said to realize a bipartite antagonistic time-varying formation tracking if the states $x_i(t)$ of the N followers meet the following conditions:

$$\lim_{t\to\infty}||x_i(t)-\sigma_i x_0(t)-h_i(t)||=0,\ i\in\{1,2\dots,N\} \tag{5}$$

Remark 1. According to Defintion 3 and Definition 5, one can obtain that when $h_i(t)\equiv 0$, the multi-agent system realizes a bipartite antagonistic formation tracking which can also be regarded as achieving a bipartite tracking consensus. In this case, the bipartite antagonistic formation tracking problem and bipartite tracking consensus problem are equivalent. More generally, the bipartite tracking consensus problem can be treated as a special case of bipartite antagonistic formation tracking problem.

3 Time-varying formation tracking protocol

For the N followers, in order to realize the bipartite antagonistic formation tracking defined by formation vector $h(t)$, one considers the following control protocol:

$$\begin{cases}u_i(t)=K_1\left(x_i(t)-\sigma_i x_0(t)\right)+c_1 S(t)+c_2\mathrm{sgn}(S(t))\\ S(t)=K_2\Big[a_{i0}(x_i(t)-h_i(t))-\sigma_i x_0(t)\\ \qquad+\sum_{j=1}^N\left(|a_{ij}|(x_i(t)-h_i(t))\right.\\ \qquad\left.-a_{ij}(x_j(t)-h_j(t))\right)\Big]\end{cases} \tag{6}$$

where $i\in\{1,2,\dots,N\}$, $c_1>0, c_2>0$ and $K_1,K_2\in\mathbb{R}^{s\times n}$ are two control parameter matrix.

Inserting protocol (6) into the equation of system Eq.(1), the following equation can be obtained:

$$\begin{cases}\dot{x}_i(t)=(A+BK_1)x_i(t)-BK_1\sigma_i x_0(t)\\ \qquad+Bc_1S(t)+Bc_2\mathrm{sgn}(S(t))\\ S(t)=K_2\Big[a_{i0}(x_i(t)-h_i(t))-\sigma_i x_0(t)\\ \qquad+\sum_{j=1}^N\left(|a_{ij}|(x_i(t)-h_i(t))\right.\\ \qquad\left.-a_{ij}(x_j(t)-h_j(t))\right)\Big]\end{cases} \tag{7}$$

Furthermore, let $\Xi=\mathrm{diag}(a_{10},a_{20},\dots,a_{N0})$, $x(t)=[x_1(t)^T,x_2(t)^T,\dots,x_N(t)^T]^T$, one has,

$$\begin{aligned}\dot{x}(t)=&(I_N\otimes(A+BK_1))x(t)\\ &+(c_1L_\Xi\otimes BK_2)(x(t)-h(t))\\ &-(\Pi\otimes BK_1 1_N+\Xi\Pi\otimes Bc_1K_2 1_N)x_0(t)\\ &+c_2(I_N\otimes B)\,\mathrm{sgn}\big((L_\Xi\otimes K_2)(x(t)-h(t))\\ &-\Xi\Pi\otimes K_2 1_N x_0(t)\big)\end{aligned} \tag{8}$$

where Π is as defined in Lemma 1 and L_Ξ is defined as $L_\Xi=L+\Xi$.

Denote $\eta(t)=[\eta_1(t)^T,\eta_2(t)^T,\dots,\eta_N(t)^T]^T$, with $\eta_i(t)=x_i(t)-h_i(t)-\sigma_i x_0(t)$, $i\in\{1,2,\dots,N\}$. Then one has $x(t)=h(t)+\eta(t)+\Pi 1_N x_0(t)$ and $\dot{x}(t)=\dot{h}(t)+\dot{\eta}(t)+\Pi 1_N\dot{x}_0(t)$. One notices that $\dot{x}_0(t)=Ax_0(t)+Bu_0(t)$, so Eq.(8) can be changed to:

$$\begin{cases}\dot{\eta}(t)=\Gamma_1(t)+\Gamma_2(t)+\Gamma_3(t)+\Gamma_4(t)+\Gamma_5(t)\\ \Gamma_1(t)=(I_N\otimes(A+BK_1)+c_1L_\Xi\otimes BK_2)\eta(t)\\ \qquad+c_2(I_N\otimes B)\mathrm{sgn}((L_\Xi\otimes K_2)\eta(t))\\ \qquad-(\Pi 1_N\otimes B)u_0(t)\\ \Gamma_2(t)=(I_N\otimes(A+BK_1))h(t)-\dot{h}(t)\\ \Gamma_3(t)=((I_N\otimes(A+BK_1))\Pi 1_N)x_0(t)-(\Pi 1_N\otimes A)x_0(t)\\ \qquad-(\Pi\otimes BK_1 1_N)x_0(t)\\ \Gamma_4(t)=((c_1L_\Xi\otimes BK_2)\Pi 1_N)x_0(t)\\ \qquad-(\Xi\Pi\otimes Bc_1K_2 1_N)x_0(t)\\ \Gamma_5(t)=c_2((I_N\otimes B)\mathrm{sgn}((L_\Xi\otimes K_2)\Pi 1_N)x_0(t)\\ \qquad-(\Xi\Pi\otimes K_2 1_N)x_0(t))\end{cases} \tag{9}$$

Since $\Pi=\mathrm{diag}(\sigma_1,\sigma_2,\dots,\sigma_N)$, $\sigma_i\in\{\pm1\}$, $\Xi=\mathrm{diag}(a_{10},a_{20},\dots,a_{N0})$ and $\sigma_i\sigma_j a_{ij}\geq 0$ for $i,j\in\{1,2,\dots,N\}$, one can get that $a_{ij}\sigma_i=|a_{ij}|\sigma_j$ and $a_{ij}\sigma_j=|a_{ij}|\sigma_i$, then:

$$\begin{aligned}&(L_\Xi\otimes K_2)\Pi 1_N x_0(t)-\Xi\Pi\otimes K_2 1_N x_0(t)\\ &=\sum_{i=1}^N\left(\sum_{j=1\neq i}^N K_2(|a_{ij}|+a_{i0})\sigma_i x_0(t)\right.\\ &\quad\left.-\sum_{j=1\neq i}^N K_2 a_{ij}\sigma_j x_0(t)-\sum_{j=1\neq i}^N K_2 a_{i0}\sigma_i x_0(t)\right)\\ &=\sum_{i=1}^N\left(\sum_{j=1\neq i}^N K_2(|a_{ij}|\sigma_i-a_{ij}\sigma_j)\right)x_0(t)\\ &=0\end{aligned} \tag{10}$$

So one can obtain that $\Gamma_5(t)=0$ in Eq.(9). Similarly, one can prove that $\Gamma_3(t)=0$ and $\Gamma_4(t)=0$. Then Eq.(9) can be simplified as:

$$\begin{cases}\dot{\eta}(t)=\Gamma_1(t)+\Gamma_2(t)\\ \Gamma_1(t)=(I_N\otimes(A+BK_1)+c_1L_\Xi\otimes BK_2)\eta(t)\\ \qquad+c_2(I_N\otimes B)\mathrm{sgn}((L_\Xi\otimes K_2)\eta(t))\\ \qquad-(\Pi 1_N\otimes B)u_0(t)\\ \Gamma_2(t)=(I_N\otimes(A+BK_1))h(t)-\dot{h}(t)\end{cases} \tag{11}$$

Theorem 1. Under protocol (6), the multi-agent system (1) can achieve a bipartite antagonistic time-varying formation tracking $h(t)$, if the two following conditions are simultaneously satisfied:

(i) For the constant matrix K_1,

$$\dot{h}_i(t)-(A+BK_1)h_i(t)=0 \tag{12}$$

(ii) One has that $c_1\geq 1/\lambda_{min}(\bar{L})$, $c_2\geq u_{max}$ and matrix $K_2=-B^TQ$, where the symmetric positive definite matrix Q is obtained by solving the following Linear Matrix Inequality (LMI),

$$Q(A+BK_1)+(A+BK_1)^TQ-2QBB^TQ<0 \tag{13}$$

Proof. If the condition (i) is holds, one gets that $\Gamma_2(t) = (I_N \otimes (A+BK_1))h(t) - \dot{h}(t) = 0$ in Eq.(11), then Eq.(11) can be rewritten as:

$$\begin{aligned}\dot{\eta}(t) = &(I_N \otimes (A+BK_1) + c_1 L_\Xi \otimes BK_2)\eta(t) \\ &+ c_2(I_N \otimes B)\mathrm{sgn}((L_\Xi \otimes K_2)\eta(t)) \\ &- (\Pi 1_N \otimes B)u_0(t) \end{aligned} \tag{14}$$

Denote $\bar{\eta}(t) = (\Pi \otimes I_N)\eta(t)$. Since $\Pi^{-1} = \Pi$, one has $\eta(t) = (\Pi \otimes I_N)\bar{\eta}(t)$ and $\dot{\eta}(t) = (\Pi \otimes I_N)\dot{\bar{\eta}}(t)$. Eq.(14) can be transformed as:

$$\begin{aligned}\dot{\bar{\eta}}(t) = &(\Pi \otimes I_N)^{-1} c_2 (I_N \otimes B)\mathrm{sgn}((L_\Xi \otimes K_2)(\Pi \otimes I_N)\bar{\eta}(t)) \\ &+ (\Pi \otimes I_N)^{-1}(I_N \otimes (A+BK_1) \\ &+ c_1 L_\Xi \otimes BK_2)(\Pi \otimes I_N)\bar{\eta}(t) \\ &- (\Pi \otimes I_N)^{-1}(\Pi 1_N \otimes B)u_0(t) \end{aligned} \tag{15}$$

Since $\Pi^{-1} = \Pi$, $(\Pi \otimes I_N)^{-1} = (\Pi \otimes I_N)$, and $\Pi \mathrm{sgn}(z) = \mathrm{sgn}(\Pi z)$, where z is an arbitrary dimension matching matrix, one has,

$$\begin{aligned}\dot{\bar{\eta}}(t) = &(I_N \otimes (A+BK_1) + c_1 \Pi L_\Xi \Pi \otimes BK_2)\bar{\eta}(t) \\ &+ c_2(I_N \otimes B)\mathrm{sgn}((\Pi L_\Xi \Pi \otimes K_2)\bar{\eta}(t)) \\ &- (1_N \otimes B)u_0(t) \end{aligned} \tag{16}$$

$\Pi \Xi \Pi = \Xi$ because of $\mathrm{diag}(a_{10}\sigma_1^2, a_{20}\sigma_2^2, \ldots, a_{N0}\sigma_N^2) = \mathrm{diag}(a_{10}, a_{20}, \ldots, a_{N0})$. $\Pi L_\Xi \Pi = \Pi L \Pi + \Pi \Xi \Pi = \Pi L \Pi + \Xi$. Denote $\bar{L} = \Pi L_\Xi \Pi = \Pi L \Pi + \Xi$, one can get that

$$\begin{aligned}\dot{\bar{\eta}}(t) = &(I_N \otimes (A+BK_1) + c_1 \bar{L} \otimes BK_2)\bar{\eta}(t) \\ &+ c_2(I_N \otimes B)\mathrm{sgn}((\bar{L} \otimes K_2)\bar{\eta}(t)) \\ &- (1_N \otimes B)u_0(t) \end{aligned} \tag{17}$$

Let

$$\psi(t) = (\bar{L} \otimes I_N)\bar{\eta}(t) \tag{18}$$

It can be thus derived from (17) that

$$\begin{aligned}\dot{\psi}(t) = &(I_N \otimes (A+BK_1) + c_1 \bar{L} \otimes BK_2)\psi(t) \\ &+ c_2(\bar{L} \otimes B)\mathrm{sgn}(\Theta(t)) \\ &- (\bar{L} 1_N \otimes B)u_0(t) \end{aligned} \tag{19}$$

where $\Theta(t) = \left[\Theta_1(t)^T, \Theta_2(t)^T, \ldots, \Theta_N(t)^T\right]^T = (I_N \otimes K_2)\psi(t)$ with $\Theta_i(t) \in \mathbb{R}^s$ for $i \in \{1, 2, \ldots, N\}$.

Lyapunov function $V(t) = \psi(t)^T (I_N \otimes Q)\psi(t)$ is selected for system (19), where $Q > 0$ is a symmetric matrix.

$$\begin{aligned}\dot{V}(t) = &2\psi^T(t)(I_N \otimes Q)\dot{\psi}(t) \\ &2\psi^T(t)(I_N \otimes Q(A+BK_1) + 2c_1 \bar{L} \otimes QBK_2)\psi(t) \\ &- 2c_2 \Theta^T(t)(\bar{L} \otimes I_s)\mathrm{sgn}(\Theta(t)) \\ &+ 2\Theta^T(t)(\bar{L} 1_N \otimes I_s)u_0(t) \end{aligned} \tag{20}$$

$\dot{V}(t)$ can be rewritten as following:

$$\begin{cases}\dot{V}(t) = V_1 + V_2 + V_3 \\ V_1 = \psi^T(t)(I_N \otimes (Q(A+BK_1) + (A+BK_1)^T Q) \\ \quad -c_1 \bar{L} \otimes 2QBB^T Q)\psi(t) \\ V_2 = -2c_2 \Theta(t)^T (\bar{L} \otimes I_s)\mathrm{sgn}(\Theta(t)) \\ V_3 = 2\Theta(t)^T (\bar{L} 1_N \otimes I_s)u_0(t) \end{cases} \tag{21}$$

Since $\bar{L} = \Pi L_\Xi \Pi = \Pi L \Pi + \Xi$, one has that,

$$\begin{aligned}&\Theta(t)^T (\bar{L} \otimes I_s)\mathrm{sgn}(\Theta(t)) \\ &= \Theta(t)^T (\Pi L \Pi \otimes I_s)\mathrm{sgn}(\Theta(t)) + \Theta(t)^T (\Xi \otimes I_s)\mathrm{sgn}(\Theta(t)) \\ &= \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \sigma_i \sigma_j a_{ij} \left(||\Theta_i(t)||_1 - \Theta_i(t)^T \mathrm{sgn}(\Theta_j(t)\right) \\ &\quad + \sum_{i=1}^{N} a_{i0} ||\Theta_i(t)||_1 \\ &\geq \sum_{i=1}^{N} a_{i0} ||\Theta_i(t)||_1 \end{aligned} \tag{22}$$

where $||\Theta_i(t)||_1 - \Theta_i(t)^T \mathrm{sgn}(\Theta_j(t) \geq 0$ for $i, j \in \{1, 2, \ldots, N\}$ because of $\sigma_i^2 = 1$, $|a_{ij}| = \sigma_i \sigma_j a_{ij}$ and $\Theta_i(t)^T \mathrm{sgn}(\Theta_i(t)) = ||\Theta_i(t)||_1$.

$$\begin{aligned}&\Theta(t)^T (\bar{L} 1_N \otimes I_s)u_0(t) \\ &= \Theta(t)^T (\Pi L \Pi 1_N \otimes I_s)u_0(t) + \Theta(t)^T (\Xi 1_N \otimes I_s)u_0(t) \\ &= \sum_{i=1}^{N} (a_{i0}\Theta_i(t))\, u_0(t) \quad \text{for} \quad \sigma_i^2 = 1, \quad \sigma_i \sigma_j a_{ij} = |a_{ij}| \\ &\leq \sum_{i=1}^{N} a_{i0} u_{max} ||\Theta_i(t)||_1 \end{aligned} \tag{23}$$

One can obtain that

$$\begin{aligned}&V_2 + V_3 \\ &= 2\Theta^T(t)(\bar{L} 1_N \otimes I_s)u_0(t) - 2c_2 \Theta^T(t)(\bar{L} \otimes I_s)\mathrm{sgn}(\Theta(t)) \\ &\leq \sum_{i=1}^{N} a_{i0}(u_{max} - c_2)||\Theta_i(t)||_1 \\ &\leq 0 \end{aligned} \tag{24}$$

Eq.(21) can be rewritten as follows:

$$\begin{aligned}\dot{V}(t) \leq &\psi^T(t)(I_N \otimes (Q(A+BK_1) + (A+BK_1)^T Q) \\ &- c_1 \bar{L} \otimes 2QBB^T Q)\psi(t) \end{aligned} \tag{25}$$

If the condition (ii) holds, one has that $K_2 = -B^T Q$. In addition $c_1 \geq 1/\lambda_{min}(\bar{L})$, one has $I_N \leq c_1 \bar{L}$, then

$$\begin{aligned}\dot{V}(t) \leq &\psi^T(t)(I_N \otimes \Big((Q(A+BK_1) + (A+BK_1)^T Q) \\ &- 2QBB^T Q)\Big)\psi(t) \\ &< 0 \end{aligned} \tag{26}$$

According to Lyapunov stability criterion, $V(t) > 0$, $\dot{V}(t) < 0$, so $||\psi(t)||$ converges to 0. Since $\psi(t) = (\bar{L} \otimes I_N)(\Pi \otimes I_N)\eta(t)$, it follows immediately that:

$$\lim_{t\to\infty} ||x_i(t) - h_i(t) - \sigma_i x_0(t)|| = \lim_{t\to\infty} ||\eta_i(t)|| = 0. \tag{27}$$

□

4 Algorithm of protocol design

The protocol (6) can be designed by the following steps:

Step 1: For all $i \in \{1, 2, \ldots, N\}$, check the following formation feasibility condition. For the formation vector $h(t)$, if there is a constant matrix K_1 satisfying the expression (28), continue; Otherwise the multi-agent system cannot form the desired formation $h(t)$ under protocol (6), stop.

$$\dot{h}_i(t) - (A + BK_1)h_i(t) = 0 \tag{28}$$

Step 2: Solve the following LMI (29) to obtain symmetric positive definite matrix Q and K_2 can be calculated by $K_2 = -B^T Q$. Choose the appropriate c_1 and c_2 such that $c_1 \geq 1/\lambda_{min}(\bar{L})$ and $c_2 \geq u_{max}$.

$$Q(A + BK_1) + (A + BK_1)^T Q - 2QBB^T Q < 0 \tag{29}$$

Remark 2. Condition (28) shows that K_1 can be used to extend the feasible set of formations. If $K_1 = 0$, the feasible set of formation becomes $\dot{h}_i(t) - Ah_i(t) = 0$, which means that only a limited set of feasible formations can be determined based on the characteristics of the system matrix A.

5 Simulation

Consider a multi-agent system with one leader and five followers which move in the X−Y plane. Dynamics equation of each agent is as following:

$$\dot{\tilde{x}}_i(t) = \tilde{A}\tilde{x}_i(t) + \tilde{B}u_i(t), \tag{30}$$

where $\tilde{x}_i = \in \mathbb{R}^4, \quad i \in \{0, 1, \ldots, 5\}$,

$$\tilde{x}_i(t) = \begin{bmatrix} p_{xi}(t) \\ v_{xi}(t) \\ p_{yi}(t) \\ v_{yi}(t) \end{bmatrix}, \tilde{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tilde{B} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$p_{xi}(t) \in \mathbb{R}$ and $p_{yi}(t) \in \mathbb{R}$ are the position of agent in the x and y directions, respevtively, and $v_{xi}(t) \in \mathbb{R}$ and $v_{yi}(t) \in \mathbb{R}$ are the velocities of agent in the x and y directions, respevtively.

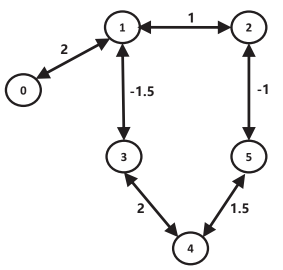

Fig. 1: Antagonistic topology of the multi-agent system.

The leader makes a circular motion around the point $(3, 3)$ with $\omega_L = 0.2\ rad/s$ and $r_L = 2\ m$. $u_0(t) = [-\omega_L^2 * r_L * cos(\omega_L t), -\omega_L^2 * r_L * sin(\omega_L t)]^T$, one can obtain that $||u_0(t)|| \leq u_{max} = 0.08$. The desired formation tracking form for the followers is two identical circles with $\omega = 1\ rad/s$ and $r = 0.5\ m$, the formation tracking vector $h(t)$ is as following,

$$h_i(t) = \begin{bmatrix} rcos(\omega t + (i-1)/\pi) \\ -\omega rsin(\omega t + (i-1)/\pi) \\ rsin(\omega t + (i-1)/\pi) \\ \omega rcos(\omega t + (i-1)/\pi) \end{bmatrix}, i \in \{1, 2\}$$

$$h_j(t) = \begin{bmatrix} rcos(\omega t + 2(j-1)/3\pi) \\ -\omega rsin(\omega t + 2(j-1)/3\pi) \\ rsin(\omega t + 2(j-1)/3\pi) \\ \omega rcos(\omega t + 2(j-1)/3\pi) \end{bmatrix}, j \in \{3, 4, 5\}$$

The antagonistic interaction topology is shown in Fig.1. One can verify that the bigraph corresponding to the topology of followers is structually balanced. Followers can be separated into two parts: $\mathcal{V}_1 = \{1, 2\}$ and $\mathcal{V}_2 = \{3, 4, 5\}$, then one has $\Pi = \text{diag}(1, 1, -1, -1, -1)$. The Laplacian matrix L is

$$L = \begin{bmatrix} 2.5 & -1 & 1.5 & 0 & 0 \\ -1 & 2 & 0 & 0 & 1 \\ 1.5 & 0 & 3.5 & -2 & 0 \\ 0 & 0 & -2 & 3.5 & -1.5 \\ 0 & 1 & 0 & -1.5 & 2.5 \end{bmatrix}$$

Matrix K_1 is calulated as $K_1 = \begin{bmatrix} -\omega^2 & 0 & 0 & 0 \\ 0 & 0 & -\omega^2 & 0 \end{bmatrix}$, then $K_1 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$. Matrix $K_2 = -\tilde{B}^T Q$ is calucated by solving the equivalent LMI: $Q^{-1}(\tilde{A} + \tilde{B}K_1)^T + (\tilde{A} + \tilde{B}K_1)Q^{-1} - 2\tilde{B}\tilde{B}^T < 0$. By using the Matlab YALMIP solver, one gets that

$$Q = \begin{bmatrix} 1.252 & -0.304 & 0 & 0 \\ -0.304 & 1.735 & 0 & 0 \\ 0 & 0 & 1.252 & -0.304 \\ 0 & 0 & -0.304 & 1.735 \end{bmatrix}$$

and $K_2 = \begin{bmatrix} -0.146 & -0.602 & 0 & 0 \\ 0 & 0 & -0.146 & -0.602 \end{bmatrix}$. In addition, one selects that $c_1 = 10$ and $c_2 = 1$. The initial states of each agent are set as:

$$\tilde{x}_0(0) = \begin{bmatrix} 1 \\ 0.5 \\ 1 \\ -0.6 \end{bmatrix}, \tilde{x}_1(0) = \begin{bmatrix} 2 \\ 1 \\ -2 \\ 0.8 \end{bmatrix}, \tilde{x}_2(0) = \begin{bmatrix} -2 \\ 0.6 \\ -1 \\ -1 \end{bmatrix}$$

$$\tilde{x}_3(0) = \begin{bmatrix} -3 \\ 1.4 \\ 0 \\ 0.1 \end{bmatrix}, \tilde{x}_4(0) = \begin{bmatrix} -3 \\ -1 \\ 2 \\ 0.9 \end{bmatrix}, \tilde{x}_5(0) = \begin{bmatrix} 0 \\ 1 \\ 4 \\ 0.7 \end{bmatrix}$$

Fig.2 and Fig.3 display the positions $p_{xi}(t)$ and $p_{yi}(t)$ of the agents in the X−Y plane within $t = 80s$ and the snapshot of those positions at $t = 80s$, respectively, where the initial positions are marked by the diamond markers and the positions at $t = 80s$ are marked by the asteriskF markers.

It can be obtained from Fig.2 and Fig.3 that the followers 1 and 2 track the leader while form a circle around the leader with radius $r = 0.5m$ and angular velocity $\omega = 1\,rad/s$, and followers 3, 4, and 5 track the leader at the symmetrical position relative to the origin and form a circle with same radius and angular velocity. Therefore, one can get that the desired bipartite antagonistic time-varying formation tracking is achieved under the protocol (6).

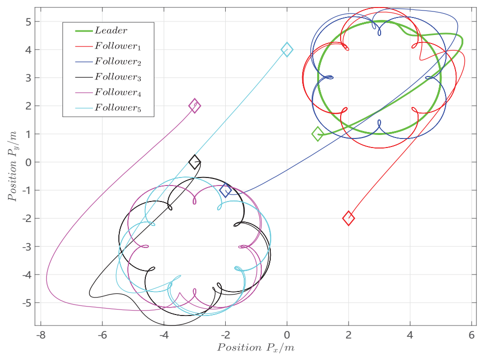

Fig. 2: Positions $p_{xi}(t)$ and $p_{yi}(t)$ in X−Y plane within $t = 80s$.

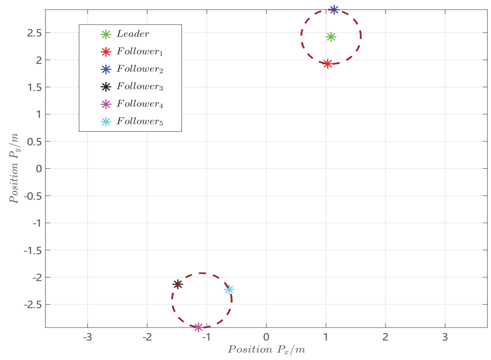

Fig. 3: Positions $p_{xi}(t)$ and $p_{yi}(t)$ in X−Y plane at $t = 80s$.

6 Conclusion

Bipartite antagonistic time-varying formation tracking problem were investigated in this paper. Sufficient conditions for multi-agent system to realize bipartite antagonistic formation tracking was given. A distributed bipartite antagonistic formation tracking protocol was proposed by using the neighboring states information and an algorithm to design the protocol was proposed. Furthermore, an approach to extend the feasible set of formation was introduced. Simulation results demonstrated that the theorical results were effective for the bipartite formation tracking problem. Based on the above analysis, it is significant to study the bipartite antagonistic time-varying formation tracking problem with switching topology in future research.

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