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# Time-varying formation of second-order discrete-time multi-agent systems under non-uniform communication delays and switching topology with application to UAV formation flying

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**Abstract:** In this study, the time-varying formation control problem for the second-order discrete-time multi-agent systems is investigated, where both the non-uniform communication time-delays and switching topology are taken into account. A linear discrete-time formation protocol is developed based on the neighbouring relative position and velocity information. Using the state transformation method and properties of the stochastic matrix, the formation feasibility condition is given and sufficient conditions for the discrete-time multi-agent systems to accomplish the time-varying formation are established. An unmanned aerial vehicles (UAVs) formation experiment platform is constructed. Using four quadrotor UAVs, UAV formation flying experiments are performed to verify the effectiveness and reliability of the discrete-time formation protocol. The experimental results show that the theoretical results can be used to deal with the time-varying formation control problem for multiple UAVs system with communication delays and switching topology.

## 1 Introduction

Formation control research, which is an important branch of the coordination control of multi-agent systems, has attracted widespread attention in the past few years [1]. Through the cooperation among multiple agents, multi-agent systems possess more powerful capabilities and can perform more complex tasks than multiple single-agents. Formation control technology has great application potential in many fields, such as target localisation [2], target enclosing [3], nuclear radiation detecting [4], and synchronous motion of unmanned aircrafts [5].

Formation control problems for multi-agent systems with first-order [6–8], second-order [9–12], and high-order [13] dynamics have been extensively studied. However, these researches focus on the fixed formation problem. In practical applications, it is very useful that the formation shape or the relative positions of agents can be time-varying. For instance, when the multi-agent systems need to achieve greater coverage or to avoid the obstacles, the positions and velocities of the agents need to be dynamically adjusted. Thus, the research on time-varying formation control problem of multi-agent systems is more significant. The time-varying formation problem for first-order multi-agent systems is investigated in [14], where a distributed observer-based protocol is designed. The time-varying formation control problem for multi-agent systems with first-order and second-order heterogeneous dynamics is investigated in [15], where a Lyapunov approach based on back-stepping method is developed for the synchronisation between first-order agents and second-order agents. In [16], a time-varying formation control problem for second-order multi-agent systems is investigated, where necessary and sufficient conditions for the realisation of the formation are given based on the formation feasibility constraints. A fast terminal sliding mode control is developed in [17] for second-order multi-agent systems to realise the desired time-varying formation in the finite time. In [18], a general time-varying formation control approach that does not depend on initial agents' states is introduced for multi-agent systems with non-linear dynamics. Time-varying formation control problem for high-order linear multi-agent systems under directed topology is addressed in [19], where an adaptive control protocol that only uses the relative states among agents is developed. A layered distributed finite-time estimator is developed in [20] to deal with the formation control problem. A time-varying formation control protocol based on event-triggered scheme is proposed in [21] for high-order multi-agent systems.

The above researches focus on the realisation of the time-varying formation under ideal conditions. However, time-varying formation control problems of multi-agent systems with communication delays, topology switching, or external disturbance are more common in practical applications. A time-varying formation control method for second-order multi-agent systems is developed in [22], where the non-uniform time-varying transmission delays are taken into account. Fully distributed protocols are developed in [23] for second-order multi-agent systems with communication delays to achieve a formation, where the maximum allowable delay is given. A time-delayed formation protocol that uses only the relative agents' position information is constructed in [24] to deal with the formation control problem of second-order multi-agent systems. Sufficient conditions for second-order multi-agent systems with time-varying transmission delays to complete a formation are given in [25], where both the upper and lower bounds of the variation rate of delay are considered. The time-varying formation control problem for multi-agent systems with second-order dynamics is investigated in [26], where two kinds of time delays, one affecting the velocity and the other affecting the position are taken into consideration. A formation tracking controller that uses both real-time and delay state information is developed in [27] for non-linear multi-agent systems. In [28], an identifier-based control protocol is proposed for second-order multi-agent systems with disturbances and model uncertainties to achieve the time-varying formation. In [29], the formation control problem that considers both the time-varying communication delays and stochastic changing topology for multi-agent systems with non-linear dynamics is investigated.

The aforementioned formation control problems are based on continuous-time model, where the agent states are assumed to be continuous. However, with the development and application of microcomputers, more and more digital systems and digital

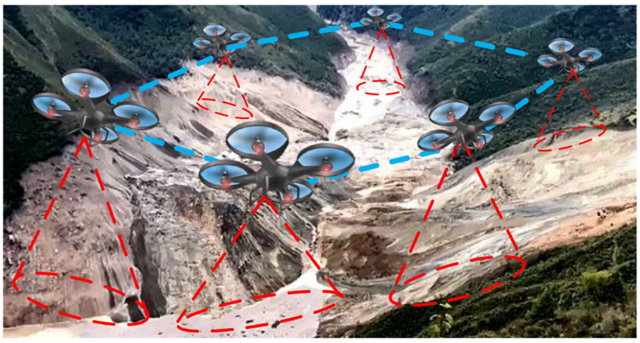

**Fig. 1** *Post-disaster assessment by multiple UAVs formation*

controllers are employed in engineering field. In this case, continuous information cannot be processed directly. Therefore, it is more interesting to study the time-varying formation control problem based on the discrete-time model and to design the discretised formation controller [30, 31]. The time-varying formation control problems for discrete-time multi-agent systems with communication delays [32–36] and switching topology [37–41] are investigated extensively. In [35], a Katz-centrality-based protocol is developed for the formation problem of second-order discrete-time multi-agent systems with time-varying transmission delay. In [36], heterogeneous time delays in the formation control problem for high-order discrete-time multi-agent systems are taken into account. In [40], the robust formation control problem for non-linear discrete-time multi-agent systems with the switching topology and the initial formation error is presented. In [41], a time-varying formation for discrete-time multi-agent systems with non-linear dynamics is completed based on an iterative learning approach. The time-varying formation control problems considering multiple constraints for discrete-time multi-agent systems are investigated in [42–47]. An iterative learning control approach for the formation problem of general discrete-time multi-agent systems with transmission delays is presented in [44], where the stochastic disturbance and structure uncertainties are considered. A non-linear projection-based approach is developed in [45] for the formation control problem of single-integrator discrete-time multi-agent systems with switching topology and communication delays. A distributed observer is constructed to deal with the formation problem of time-delayed discrete-time multi-agent systems with the connected changing topology in [46]. Sufficient conditions for heterogeneous discrete-time multi-agent systems with non-linear dynamics and structural uncertainties to realise a time-varying formation are given in [47].

Most of the above researches only perform theoretical analysis and numerical simulation. Only a few papers [14, 16, 47–50] give the experimental verification. In [14], an experiment with five wheeled mobile robots is applied to verify the observer-based formation control protocol. In [16], a formation flight experiment with five unmanned aerial vehicles (UAVs) is conducted to demonstrate the effectiveness of the time-varying formation protocol. In [47], the proposed formation controller for heterogeneous discrete-time multi-agent systems is applied on a multiple robot system with four Arduino FPV Robots. In [48], the formation protocol for second-order multi-agent systems with the switching topology is applied on a multiple UAVs system with four UAVs. In [49], experiments with six nano-quadrotors are performed to verify the theoretical results. To the best of our knowledge, the time-varying formation control problem for second-order discrete-time multi-agent systems with non-uniform communication time-delays and the switching topology is still open and there is no corresponding experimental application on UAVs.

Time-varying formation control technology of multi-agent system has many potential application scenarios, such as environmental management, disaster management [51] and post-disaster assessment [52]. A single UAV has the shortcomings of small detection area, short cruising range and low operating efficiency. In contrast, multiple UAVs formation can achieve greater coverage and improve operational efficiency. For example, after an earthquake or debris flow, multi-UAV formation can quickly and efficiently survey the situation of the disaster site, assess the severity of the disaster, and improve the efficiency of subsequent rescue work. Fig. 1 shows the scenario of utilising multiple UAV formation to survey the disaster situation after a landslide.

This paper investigates the time-varying formation control problem with non-uniform communication delays and switching topology for second-order discrete-time multi-agent systems. A discrete-time formation control protocol using relative neighbouring agents' states is proposed. By decomposing the closed-loop system, the formation feasibility condition based on the formation vector is established. Moreover, using the state transformation method and properties of the stochastic matrix, sufficient conditions for second-order discrete-time multi-agent systems to achieve the time-varying formation with communication delays and switching topology are given. A simulation is shown to demonstrate the effectiveness of the formation protocol. In order to demonstrate the application of the proposed formation protocol on multiple UAVs system, two UAV flying experiments are conducted. In the first experiment, only switching topology is considered; in the second experiment, both the switching topology and non-uniform communication delays are considered. The analyses of the experimental results are also given.

Compared with the previous research of time-varying formation control. The contributions of this paper are threefold. First, both the time-varying communication delays and topology variation are taken into account, where the communication delays can be non-uniform. In [37, 38, 40, 41], only the switching topology is considered and in [24, 27, 32, 33], only the uniformed or fixed communication delays is taken into consideration. Second, the structure of multi-agent systems and control protocol are constructed based on discrete-time model. These theoretical results can be directly applied to solve the time-varying formation control problem of discrete-time multi-agent systems. With the application of digital systems and digital controllers, discretised formation control protocols are easy to be implemented and can effectively reduce the communication consumption. However, the researches on formation control of multi-agent systems in [17–19, 22–27] are based on continuous-time model and the results cannot be directly applied on the digital controllers. Third, the proposed formation protocol can be used to deal with the time-varying formation control problem of multiple UAVs system. Using four quadrotor UAVs, two experiments are given to verify the effectiveness and reliability of the proposed formation protocol. In [6–13, 22–29], and [32, 33, 35, 44, 53], the proposed formation protocols are only verified by numerical simulations.

The remaining of this paper is organised as follows. Preliminaries of graph and matrix theory are presented and the time-varying formation problem is defined in Section 2. A linear time-varying discrete-time formation protocol is given and a theorem that ensures the realisation of formation is proven in Section 3. Numerical simulation is given in Section 4 to verify the effectiveness of the proposed protocol. In Section 5, a UAV formation platform is introduced, the experiments with four UAVs are performed, and the analyses of experimental results are given. Conclusions are drawn in Section 6.

The following notations are applied in this paper for simplicity. $\mathbb{R}^{M\times N}$ represents the set of real matrices with $M$ rows and $N$ columns. $\mathbf{1}$ represents a vector $[1,1,\ldots,1]^{\mathrm{T}}$ with an appropriate dimension. $\otimes$ is the Kronecker product. $\|\cdot\|_2$ indicates the Euclidean norm.

## 2 Preliminaries and problem description

### 2.1 Basic properties of graph theory

Let $\mathscr{G}=(\mathscr{V},\mathscr{E},\mathscr{A})$ denote a weighted directed graph of $N$ nodes with the set of nodes $\mathscr{V}=\{v_1,v_2,\ldots,v_N\}$, the set of edges $\mathscr{E}=\{\mathscr{E}_{ij}=(v_i,v_j),\ \mathscr{E}\subseteq\mathscr{V}\times\mathscr{V}\}$, and a weighted adjacency matrix $\mathscr{A}=(a_{ij})_{N\times N}$. The weighted elements $a_{ij}$ are non-negative and it is assumed that $a_{ii}=0,\ \forall i\in\{1,2,\ldots,N\}$. A graph $\mathscr{G}$ is called undirected if $a_{ij}=a_{ji},\ \forall i,j\in\{1,2,\ldots,N\}$.

$\mathcal{N}_j = \{v_j \in \mathcal{V}, \mathcal{E}_{ji} = (v_j, v_i) \in \mathcal{E}\}$ represents the set of neighbour of node $v_j$. If there exists a series of edges $(v_i, v_{i_1}), (v_{i_1}, v_{i_2}), \ldots, (v_{i_l}, v_j)$ with $v_{i_k}(k = 1, 2, \ldots, l)$ different nodes of the graph, then it called that there exists a directed path between nodes $v_i$ and $v_j$. A directed graph $\mathcal{G}$ is said to contain a directed spanning tree if there exists at least one node that has directed paths to all the other nodes. The in-degree matrix $\mathcal{W}$ is defined as $\mathcal{W} = \mathrm{diag}(\deg_{\mathrm{in}}(v_1), \deg_{\mathrm{in}}(v_2), \ldots, \deg_{\mathrm{in}}(v_N))$, where the in-degree of node $v_i$ is $\deg_{\mathrm{in}}(v_i) = \sum_{j=1, j \neq i}^{N} a_{ij}$. The Laplacian matrix $L$ of graph $\mathcal{G}$ is defined as $L = \mathcal{W} - \mathcal{A}$. For a directed graph $\mathcal{G}$, 0 is an eigenvalue of Laplacian matrix $L$ and $\mathbf{1}$ is the associated right eigenvector, i.e. $L\mathbf{1} = 0$.

The topology considered in this paper is dynamically changing. The topology of the graph $\mathcal{G}$ at time $k$ is denoted by $\mathcal{G}(k)$. Let $\Omega_N = \{\mathcal{G}(k), k \in \mathbb{Z}^+\}$ be the set of all the possible topologies of graph $\mathcal{G}$. Let $\mathcal{N}_i(k)$ represent the set of neighbour of node $i$ at time $k$. Let $L(k) = (l_{ij}(k))_{N \times N}$ denote the Laplacian matrix of the graph corresponding to the topology $\mathcal{G}(k)$. For all possible $L(k)$, let $d_{\max}$ be the largest diagonal entry of $L(k)$. The communication delay among the agents is described by $\tau_{ij} \in \mathbb{R}, i \neq j$, which represents the time-delay from agent $j$ to $i$. Assume that the communication delays are bounded, namely, $\tau_{ij} \leq \tau_{\max}$, where $\tau_{\max}$ is the maximal delay.

*Lemma 1:* Consider a square matrix $M = (m_{ij})_{n \times n} \in \mathbb{R}^{n \times n}$,

(i) $M$ is called a stochastic matrix if all the elements of $M$ are non-negative and for $\forall i \in \{1, 2, \ldots, n\}$, $\sum_{j=1}^{n} m_{ij} = 1$.
(ii) If $M$ is a stochastic matrix and there exists a constant vector $c \in \mathbb{R}^n$ such that $\prod_{j=1}^{+\infty} M^j = \mathbf{1}c^{\mathrm{T}}$, then $M$ is called stochastic indecomposable and aperiodic (SIA) matrix [54].

### *2.2 Definition of time-varying formation*

Consider a discrete-time multi-agent system with $N$ agents. The dynamic equation of agent $i$ is modelled as

$$\begin{aligned} p_i((k+1)T) &= p_i(kT) + Tv_i(kT) \\ v_i((k+1)T) &= v_i(kT) + Tu_i(kT) \end{aligned} \tag{1}$$

where $k \in \mathbb{Z}^+$, $T \in \mathbb{R}$ is the sample period, and $u_i(kT) \in \mathbb{R}^n$ is the control input of agent $i$ at time $kT$. $p_i(kT) \in \mathbb{R}^n$ and $v_i(kT) \in \mathbb{R}^n$ are the position and velocity of agent $i$ at time $kT$, respectively.

In the following, let $n = 1$ for the sake of simplicity in the description. However, using Kronecker product, all the results hereafter can be directly extended to the higher dimensional cases.

Denote $x_i(kT) = [p_i(kT), v_i(kT)]^{\mathrm{T}} \in \mathbb{R}^2$ and replace $kT$ by $k$ for simplicity. Then multi-agent system (1) can be transformed to

$$x_i(k+1) = Ax_i(k) + Bu_i(k) \tag{2}$$

where $i = 1, 2, \ldots, N$ and

$$A = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ T \end{bmatrix}$$

*Remark 1:* Multi-agent system (2) is a discrete state space representation of a double integrator system. Most UAVs and unmanned ground vehicles (UGVs) can be modelled as a double integrator system after linearisation [48].

*Definition 1:* The multi-agent system (2) realises consensus if there exists a $f_R(k) \in \mathbb{R}^2$ such that the following equation is satisfied:

$$\lim_{k \to +\infty} (x_i(k) - f_R(k)) = \mathbf{0} \quad \forall i = 1, 2, \ldots, N \tag{3}$$

where $f_R(k)$ is a vector-valued function and is defined as the consensus reference function.

Denote $x(k) = [x_1(k)^{\mathrm{T}}, x_2(k)^{\mathrm{T}}, \ldots, x_N(k)^{\mathrm{T}}]^{\mathrm{T}}$, and let $h(k) = [h_1(k)^{\mathrm{T}}, h_2(k)^{\mathrm{T}}, \ldots, h_N(k)^{\mathrm{T}}]^{\mathrm{T}}$ be the desired formation vector for system (2), where $h_i(k) = [h_{ip}(k), h_{iv}(k)]^{\mathrm{T}} \in \mathbb{R}^2$ is the formation vector for agent $i$.

*Definition 2:* The multi-agent system (2) realises the time-varying formation $h(k)$ if there exists a $h_R(k) \in \mathbb{R}^2$ such that the following condition is met:

$$\lim_{k \to +\infty} (x_i(k) - h_i(k) - h_R(k)) = \mathbf{0} \quad \forall i = 1, 2, \ldots, N \tag{4}$$

where $h_R(k)$ is a vector-valued function and is defined as the formation reference function.

*Remark 2:* According to Definitions 1 and 2, one can obtain that if $h_i(k) \equiv \mathbf{0}$, realisations of consensus and time-varying formation $h_i(k)$ for the multi-agent system (2) are equivalent. In this case, the consensus reference function $f_R(k)$ and formation reference function $h_R(k)$ are the same. More generally, the consensus problem can be considered as a particular case of the time-varying formation problem to be dealt with.

## 3 Time-varying formation protocol

For multi-agent system (2) with non-uniform communication delays and switching topology, in order to achieve the formation defined by vector $h(k)$, the following discrete-time control protocol is proposed:

$$\begin{aligned} u_i(k) = {} & K_1(x_i(k) - h_i(k)) + h_{ia}(k) \\ & + K_2 \sum_{j \in \mathcal{N}_i(k)} a_{ij}(k)(x_j(k_\tau) - h_j(k_\tau) - x_i(k) + h_i(k)) \end{aligned} \tag{5}$$

where $i = 1, 2, \ldots, N$, $k_\tau = k - \tau_{ij}$,

$$h_{ia}(k) = (h_{iv}(k+1) - h_{iv}(k))/T \tag{6}$$

$K_1 = [\bar{k}_{11}, \bar{k}_{12}] \in \mathbb{R}^{1 \times 2}$ and $K_2 = [\bar{k}_{21}, \bar{k}_{22}] \in \mathbb{R}^{1 \times 2}$ are two control parameter matrices.

Under the protocol (5), the multi-agent system (2) can be described as follows:

$$\begin{aligned} x_i(k+1) = {} & (A + BK_1)x_i(k) + B(h_{ia}(k) - K_1 h_i(k)) \\ & + BK_2 \sum_{j \in \mathcal{N}_i(k)} a_{ij}(k)(x_j(k_\tau) - h_j(k_\tau) - x_i(k) + h_i(k)) \end{aligned} \tag{7}$$

Let $\varepsilon_i(k) = x_i(k) - h_i(k) = [\varepsilon_{ip}(k), \varepsilon_{iv}(k)]^{\mathrm{T}}$, then (7) is transformed to

$$\begin{aligned} \varepsilon_i(k+1) = {} & (A + BK_1)\varepsilon_i(k) \\ & + BK_2 \sum_{j \in \mathcal{N}_i(k)} a_{ij}(k)(\varepsilon_j(k_\tau) - \varepsilon_i(k)) \\ & + Ah_i(k) + Bh_{ia}(k) - h_i(k+1) \end{aligned} \tag{8}$$

Define the following subsystem for each agent $i$,

$$\begin{aligned} \varepsilon_i(k+1) = {} & (A + BK_1)\varepsilon_i(k) \\ & + BK_2 \sum_{j \in \mathcal{N}_i(k)} a_{ij}(k)(\varepsilon_j(k_\tau) - \varepsilon_i(k)) \end{aligned} \tag{9}$$

and the following lemma can be obtained directly.

*Lemma 2:* For each agent $i$, the closed-loop system (8) is stable if $Ah_i(k) + Bh_{ia}(k) - h_i(k+1) = \mathbf{0}$ and the subsystem (9) is stable.

Since only components of the formation vector $h_i(k) = [h_{ip}(k), h_{iv}(k)]^{\mathrm{T}}$ are involved in

$Ah_i(k) + Bh_{ia}(k) - h_i(k+1) = \mathbf{0}$, it is called a **formation feasibility condition**. Considering the forms of matrices $A$, $B$, and (6), the following more compact formation feasibility condition can be obtained:

$$h_{ip}(k+1) = h_{ip}(k) + Th_{iv}(k), \quad i = 1, 2, \ldots, N \tag{10}$$

*Remark 3:* The formation feasibility condition gives the condition for judging whether the desired formation $h_i(k)$ is applicable.

Denote $\varepsilon(k) = [\varepsilon_1(k)^{\mathrm{T}}, \varepsilon_2(k)^{\mathrm{T}}, \ldots, \varepsilon_N(k)^{\mathrm{T}}]^{\mathrm{T}}$. Assume that the formation feasibility condition is met, then from (9), one can get that

$$\begin{aligned}\varepsilon(k+1) = \Xi(k)\varepsilon(k) &+ (\Upsilon_0(k) \otimes BK_2)\varepsilon(k) \\ &+ (\Upsilon_1(k) \otimes BK_2)\varepsilon(k-1) + \cdots \\ &+ (\Upsilon_{\tau_{\max}}(k) \otimes BK_2)\varepsilon(k - \tau_{\max})\end{aligned} \tag{11}$$

where $\Xi(k) = I_N \otimes (A + BK_1) - L_d(k) \otimes BK_2$, $L_d(k)$ is a diagonal matrix consisting of diagonal elements of Laplacian matrix $L(k)$, namely, $L_d(k) = \mathrm{diag}(l_{11}(k), l_{22}(k), \ldots, l_{NN}(k))$. Besides, $m = 0, 1, \ldots, \tau_{\max}$, $\Upsilon_m(k) \in \mathbb{R}^{N \times N}$. The $(i,j)$ entry of $\Upsilon_m(k)$ is $a_{ij}$ if $m = \tau_{ij}$, is zero if not. $\Upsilon_m(k)$ represents a topological weight matrix between the agents with communication delays $m$. In addition, according to the definition of $L(k)$, $L(k) = L_d(k) - \sum_{m=0}^{\tau_{\max}} \Upsilon_m(k)$ can be obtained.

Then the following more compact equation can be obtained:

$$\varepsilon(k+1) = \Xi(k)\varepsilon(k) + \sum_{m=0}^{\tau_{\max}} (\Upsilon_m(k) \otimes BK_2)\varepsilon(k-m) \tag{12}$$

Let $\bar{\varepsilon}_i(k) = [\varepsilon_{ip}(k), \varepsilon_{ip}(k) + R_K \varepsilon_{iv}(k)]^{\mathrm{T}}$, where $R_K = \bar{k}_{22}/\bar{k}_{21}$. Denote $\bar{\varepsilon}(k) = [\bar{\varepsilon}_1(k)^{\mathrm{T}}, \bar{\varepsilon}_2(k)^{\mathrm{T}}, \ldots, \bar{\varepsilon}_N(k)^{\mathrm{T}}]^{\mathrm{T}}$, one has that $\bar{\varepsilon}(k) = (I_N \otimes P)\varepsilon(k)$, where

$$P = \begin{bmatrix} 1 & 0 \\ 1 & R_K \end{bmatrix}, \quad P^{-1} = \begin{bmatrix} 1 & 0 \\ -\dfrac{1}{R_K} & \dfrac{1}{R_K} \end{bmatrix}$$

Equation (12) can be converted to

$$\bar{\varepsilon}(k+1) = \bar{\Xi}(k)\bar{\varepsilon}(k) + \sum_{m=0}^{\tau_{\max}} (\Upsilon_m(k) \otimes \bar{B})\bar{\varepsilon}(k-m) \tag{13}$$

where $\bar{\Xi}(k) = (I_N \otimes \bar{A} - L_d(k) \otimes \bar{B})$, $\bar{A} = P(A + BK_1)P^{-1}$

$$\bar{A} = \begin{bmatrix} 1 - \dfrac{T}{R_K} & \dfrac{T}{R_K} \\ \bar{k}_{11}TR_K - (1 + \bar{k}_{12}R_K)\dfrac{T}{R_K} & 1 + (1 + \bar{k}_{12}R_K)\dfrac{T}{R_K} \end{bmatrix}$$

$$\bar{B} = PBK_2P^{-1} = \begin{bmatrix} 0 & 0 \\ 0 & \bar{k}_{22}T \end{bmatrix}$$

Let $\eta(k) = [\bar{\varepsilon}(k)^{\mathrm{T}}, \bar{\varepsilon}(k-1)^{\mathrm{T}}, \ldots, \bar{\varepsilon}(k - \tau_{\max})^{\mathrm{T}}]^{\mathrm{T}}$, then (13) can be further converted to

$$\eta(k+1) = \Gamma(k)\eta(k) \tag{14}$$

with

$$\Gamma(k) = \begin{bmatrix} \Gamma_0(k) & \Gamma_1(k) & \cdots & \Gamma_{\tau_{\max}-1}(k) & \Gamma_{\tau_{\max}}(k) \\ I_N & 0 & \cdots & 0 & 0 \\ 0 & I_N & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I_N & 0 \end{bmatrix}$$

where $\Gamma_0(k) = \bar{\Xi}(k) + \Upsilon_0(k) \otimes \bar{B}$ and for $m = 1, 2, \ldots, \tau_{\max}$, $\Gamma_m(k) = \Upsilon_m(k) \otimes \bar{B}$.

*Lemma 3:* The multi-agent system (2) realises the time-varying formation defined by the formation vector $h(k)$ if the system (14) is asymptotically stable, namely, $\lim_{k \to +\infty} \eta(k)$ exists.

*Proof:* Since $\eta(k) = [\bar{\varepsilon}(k)^{\mathrm{T}}, \bar{\varepsilon}(k-1)^{\mathrm{T}}, \ldots, \bar{\varepsilon}(k - \tau_{\max})^{\mathrm{T}}]^{\mathrm{T}}$, one has that $\lim_{k \to +\infty} \bar{\varepsilon}(k)$ exists if system (14) is asymptotically stable.

Furthermore, $\lim_{k \to +\infty} \varepsilon(k)$ exists since one has that $\varepsilon(k) = (I_N \otimes P^{-1})\bar{\varepsilon}(k)$ with a non-singular matrix $P$.

Since $\varepsilon(k) = x(k) - h(k)$, the existence of $\lim_{k \to +\infty} (x(k) - h(k))$ can be obtained, which means that the multi-agent system (2) realises the time-varying formation defined by formation vector $h(k)$. The proof is completed. □

Two lemmas will be introduced to facilitate the proof of the stability of system (14).

*Lemma 4:* If $\Gamma(k)$ is a stochastic matrix, for a period of time $[k_1, k_2]$, $k_2 > k_1$, $k_1, k_2 \in \mathbb{Z}^+$, and the union of the graphs $\bigcup_{k=k_1}^{k_2} \mathscr{G}(k)$ has a spanning tree. Then $\prod_{k=k_1}^{k_2} \Gamma(k)$ is a SIA matrix [55].

*Lemma 5:* Consider a finite set of SIA matrices $\Psi_1, \Psi_2, \ldots, \Psi_m \in \mathbb{R}^{n \times n}$ and for every series of matrices $\Psi_{j_1}, \Psi_{j_2}, \ldots, \Psi_{j_i}$ with $i > 1$, the matrix product $\Psi_{j_i}\Psi_{j_{i-1}} \cdots \Psi_{j_1}$ is a SIA matrix. Then, for every infinite sequence $\Psi_{j_1}, \Psi_{j_2}, \ldots$, a vector $c \in \mathbb{R}^n$ can be founded such that $\prod_{i=1}^{+\infty} \Psi_{j_i} = \mathbf{1}c^{\mathrm{T}}$ [54].

*Theorem 1:* The multi-agent system (2) with protocol (5) can accomplish the time-varying formation $h(k)$ if the following conditions are satisfied simultaneously:

(i) $1 + (1 + \bar{k}_{12}R_K)(T/R_K) > d_{\max}\bar{k}_{22}T$, $T < R_K$, and $1 + \bar{k}_{12}R_K < 0$ with $\bar{k}_{21} > 0$, $\bar{k}_{22} > 0$, $\bar{k}_{12} < 0$, $\bar{k}_{11} = 0$.
(ii) There exists an infinite serie of time $k_0 = 0, k_1, k_2, \ldots$ and for $m, \mu \in \mathbb{Z}^+$, $0 < k_{m+1} - k_m \le \mu$, the union of graph $\bigcup_{k=k_m}^{k_{m+1}-1} \mathscr{G}(k)$ has a spanning tree.

*Proof:* If the formation feasibility condition is satisfied, the multi-agent system (2) with protocol (5) can be transformed to system (14). From conditions i), one can get that all the entries of $\bar{A}$ are non-negative, and row sum of $\bar{A}$ is 1, so $\bar{A}$ is a stochastic matrix.

Since $L(k) = L_d(k) - \sum_{m=0}^{\tau_{\max}} \Upsilon_m(k)$ and $L\mathbf{1} = 0$, one can get that $\Gamma(k)\mathbf{1} = \mathbf{1}$. Furthermore, from the definition of $\Gamma(k)$, it is clear that all the entries of $\Gamma(k)$ are non-negative. Thus, $\Gamma(k)$ is a stochastic matrix.

Denote $m_k \in \mathbb{Z}^+$ the largest integer such that $k_{m_k} \le k$ for every $k \ge 0$. Let $\Theta(m) = \Gamma(k_{m+1} - 1)\Gamma(k_{m+1} - 2) \cdots \Gamma(k_m)$, then,

$$\eta(k+1) = \Gamma(k) \cdots \Gamma(k_{m_k}) \prod_{m=0}^{m_k - 1} \Theta(m)\eta(0)$$

Since $m, \mu \in \mathbb{Z}^+$, $0 < k_{m+1} - k_m \le \mu$, the union of graph $\bigcup_{k=k_m}^{k_{m+1}-1} \mathscr{G}(k)$ has a spanning tree, according to Lemma 4, $\Theta(m)$ is an SIA matrix. In addition, for an integer $j$, the union of graph

```
 1: Choose the desired time-varying formation h(k);
 2: if formation feasibility condition is met then
 3:    According to all the possible Laplacian matrix L(k),
       determine d_max;
 4:    Choose k̄_11 = 0 and select a common sample time T,
       such as 0.05s or 0.1s;
 5:    Depending on T < R_K, select a suitable set of k̄_21 > 0
       and k̄_22 > 0;
 6:    Choose an appropriate k̄_12 < 0;
 7:    if 1 + (1 + k̄_12 R_K) T/R_K ≤ d_max k̄_22 T then
 8:       back to Step 5;
 9:    end if
10: else
11:    back to Step 1;
12: end if
```

**Fig. 2** *Algorithm 1: Procedure to construct the time-varying formation protocol*

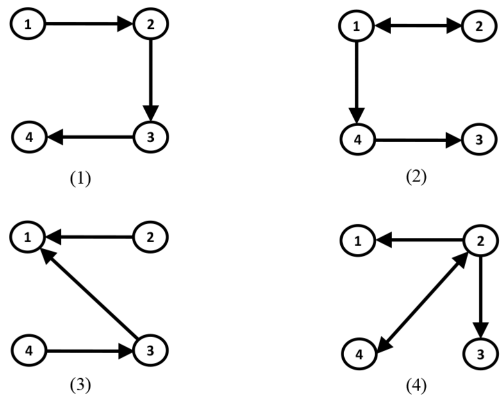

**Fig. 3** *Set of four possible topologies*

$\bigcup_{i=k_m}^{k_{m+j}-1} \mathscr{G}(i)$ has a spanning tree, thus $\prod_{s=m}^{m+j+1} \Theta(s)$ is an SIA matrix.

All the possible topologies $\mathscr{G}(k)$ form a finite set $\Omega_N$, then all the $a_{ij}(k)$ belong to a finite set. Moreover, $0 < k_{m+1} - k_m \le \mu$, thus one can obtain that all the possible $\Theta(j)$ also form a finite set. Hence, according to Lemma 5, there exists a constant vector $c \in \mathbb{R}^{2(\tau_{\max}+1)N}$ such that

$$\prod_{i=0}^{+\infty} \Theta(i) = \mathbf{1}c^{\mathrm{T}}$$

Thus, it follows that

$$\begin{aligned} \lim_{k \to +\infty} \eta(k+1) &= \lim_{k \to +\infty} (\Gamma(k) \cdots \Gamma(k_{m_k}) \prod_{m=0}^{m_k - 1} \Theta(m) \eta(0)) \\ &= \prod_{i=0}^{+\infty} \Theta(i) \eta(0) \\ &= \mathbf{1}c^{\mathrm{T}} \eta(0) \end{aligned} \tag{15}$$

Then one can obtain that $\lim_{k \to +\infty} \bar{\varepsilon}(k) = \mathbf{1}c^{\mathrm{T}}\eta(0)$, and it follows that $\lim_{k \to +\infty} (x(k) - h(k)) = \lim_{k \to +\infty} \varepsilon(k) = (I_N \otimes P^{-1})\mathbf{1}c^{\mathrm{T}}\eta(0)$. It means that the time-varying formation is realised. The proof is completed. □

*Remark 4:* The existence of communication delays and switching topologies will slow the convergence rate of the closed-loop system. More specifically, the greater the communication delay, the slower the system converges. When the communication delay between the two agents is large enough, it can be considered that there can be no information exchange between the two agents. In this case, the topology of the system has changed from the perspective of the whole multi-agent system. From the proof of Theorem 1, one can see that the time-varying formation can be also achieved if the communication delays among agents are uniform or even reduced to zero. In addition, when the topology is fixed, the time-varying formation can be realised if the topology has a spanning tree.

*Theorem 2:* If the multi-agent system (1) realises the time-varying formation $h(k)$, then the formation reference function $h_R(k)$ satisfies that

$$\lim_{k \to +\infty} h_R(k) = (I_N \otimes P^{-1})\mathbf{1}c^{\mathrm{T}}\eta(0) \tag{16}$$

where $\eta(0) = \mathbf{1} \otimes [(I_N \otimes P)(x(0) - h(0))]$.

*Proof:* From the proof of Theorem 1, one obtains that $\lim_{k \to +\infty} \eta(k+1) = \mathbf{1}c^{\mathrm{T}}\eta(0)$. In addition, $\eta(0) = [\bar{\varepsilon}(0)^{\mathrm{T}}, \bar{\varepsilon}(-1)^{\mathrm{T}}, \ldots, \bar{\varepsilon}(-\tau_{\max})^{\mathrm{T}}]^{\mathrm{T}}$. Assume that $\bar{\varepsilon}(k \le 0) = \bar{\varepsilon}(0)$, then the following initial equation can be obtained:

$$\eta(0) = \mathbf{1} \otimes \bar{\varepsilon}(0) = \mathbf{1} \otimes [(I_N \otimes P)(x(0) - h(0))] \tag{17}$$

According to Definition 2, one has that

$$\lim_{k \to +\infty} h_R(k) = \lim_{k \to +\infty} (x_i(k) - h_i(k)) \tag{18}$$

From the proof of Theorem 1, one can obtain that

$$\lim_{k \to +\infty} (x(k) - h(k)) = (I_N \otimes P^{-1})\mathbf{1}c^{\mathrm{T}}\eta(0) \tag{19}$$

Combining (17), (18), and (19), the formation reference equation (16) can be obtained. The proof is completed. □

*Remark 5:* The formation reference function determines the trajectory of the formation centre. According to Theorem 2, one can see that the formation reference function $h_R(k)$ depends on the initial states of $x(k)$ and $h(k)$.

According to Theorem 1, a feasible procedure to construct the time-varying formation protocol is given in Algorithm 1 (see Fig. 2).

## 4 Simulation

In this section, a discrete-time multi-agent system that includes four agents moving in the $X$–$Y$ plane is considered. The position and velocity of the agents in the $X$ and $Y$ directions are both taken into account. The state of each agent is defined as $x_i(k) = [p_{ix}(k), v_{ix}(k), p_{iy}(k), v_{iy}(k)]^{\mathrm{T}}, i \in \{1, 2, 3, 4\}$.

The sample time $T$ is 0.05 s. In this simulation, both the switching topology and non-uniform time-delay constraints are taken into account. Fig. 3 indicates a set of four possible interaction topologies and the corresponding topology at different times are shown in Fig. 4. The time-delays among the four agents are selected as

$$\begin{aligned} \tau_{12} = \tau_{21} = \tau_{23} = \tau_{32} = T \\ \tau_{13} = \tau_{31} = \tau_{34} = \tau_{43} = 2T \\ \tau_{14} = \tau_{41} = \tau_{13} = \tau_{31} = 3T \end{aligned}$$

The following formation error function $\Delta f(k)$ is defined to measure the achievement of the time-varying formation

$$\Delta f(k) = \| x(k) - h(k) \|_2 \tag{20}$$

According to Theorem 1, the parameters in protocol (5) are selected as $\bar{k}_{11} = -5$, $\bar{k}_{12} = -1$, $\bar{k}_{21} = 0.2$, and $\bar{k}_{22} = 0.4$. The expected time-varying formation is a circular motion, where $r = 1$ m, $\omega = 3.14$ rad/s, and the phase difference among four agents is $\pi/2$. The corresponding formation vector is as follows:

$$h_i(k) = \begin{bmatrix} r\cos(\omega k + (i-1)\pi/2) \\ -\omega r\sin(\omega k + (i-1)\pi/2) \\ r\sin(\omega k + (i-1)\pi/2) \\ \omega r\cos(\omega k + (i-1)\pi/2) \end{bmatrix}, \quad i \in \{1, 2, 3, 4\}$$

It can be verified that the formation vector $h_i(k)$ satisfies the formation feasibility conditions.

The initial states of the four agents are set to

$$x_1(0) = [0.8\,\text{m}, -0.1\,\text{m/s}, -0.4\,\text{m}, 0.5\,\text{m/s}]^{\rm T}$$
$$x_2(0) = [-0.7\,\text{m}, 0\,\text{m/s}, 0.7\,\text{m}, -0.1\,\text{m/s}]^{\rm T}$$
$$x_3(0) = [-0.5\,\text{m}, 0.5\,\text{m/s}, -0.8\,\text{m}, 0\,\text{m/s}]^{\rm T}$$
$$x_4(0) = [0.6\,\text{m}, 0.1\,\text{m/s}, -0.7\,\text{m}, 0.2\,\text{m/s}]^{\rm T}$$

Fig. 5 shows the positions of the four agents in the $X$–$Y$ plane within 40 s, where the initial positions of agents are represented by round markers and their positions at 40 s are noted by hexagon markers. Taking agent 1 as an example, the velocities and control inputs of agent 1 in the directions $X$ and $Y$ are indicated in Figs. 6 and 7, respectively. The formation error $\Delta f(k)$ is shown in Fig. 8.

According to the positions of four hexagon markers in Fig. 5, the four agents reach exactly on the circumference of the circle with $r = 1$ m at 40 s. One can see that the desired circular formation with $r = 1$ m among the four agents is accomplished. From Fig. 6, one can obtain that the formation speed of four agents is about 3.1 m/s, which is consistent with the desired formation speed calculated by the formula $v = \omega r$, where $\omega = 3.14$ rad/s and $r = 1$ m. Taking the control inputs of agent 1 in Fig. 7 as an example, it can be seen that both the control inputs in the directions $X$ and $Y$ are sine wave with a period 2 s and an amplitude 9.85 m/s$^2$. In addition, the formation error $\Delta f(k)$ in Fig. 8 decreases rapidly at the beginning and converges to zeros asymptotically, which means that the desired time-varying formation is achieved.

## 5 Application to formation control of UAVs

In this section, the proposed time-varying formation protocol is applied to deal with the formation control problem of multiple UAVs system. Two types of experiments are performed, which include one experiment with the switching topology and the other experiment with the switching topology and non-uniform time-delays. First, the UAV formation platform used in the experiment is introduced. Second, quadrotor UAV dynamics models and formation control models are established. Third, the configurations of the experiments are given and the analyses of experimental results are shown. The two experimental videos can be found at https://www.bilibili.com/video/av88058644 or https://youtu.be/GytboIQjqDg.

### *5.1 Formation platform*

The UAV formation platform, includes four parts: indoor positioning system, quadrotor UAVs, router and ground station centre (GSC). The global structure of the formation platform is shown in Fig. 9. The indoor positioning system that includes 12 high precision cameras can measure the positions and velocities of UAVs. The position measurement accuracy of the positioning system is 0.5 mm. The information transmission among the UAVs is achieved by a gigabit router. The wingspan and weight of the quadrotor UAV are 250 mm and 132 g, respectively. The maximum flight time of the quadrotor is about 10 min. Each quadrotor is equipped with a Raspberry Pi 3B+ (RP3) and a flight controller system (FCS). A three-axis digital gyroscope, a six-axis accelerometer, a magnetometer and a precision barometer are embedded in the FCS to obtain the attitudes, accelerations, and the heights of the quadrotors. The RP3 is used to analyse the GSC's control commands and generate the desired attitude information. The FCS is used to control the quadrotor to achieve the desired attitude. The GSC based on matlab are used to send takeoff and landing instructions to the quadrotors. During the formation, the quadrotors do not need to receive the control command of the

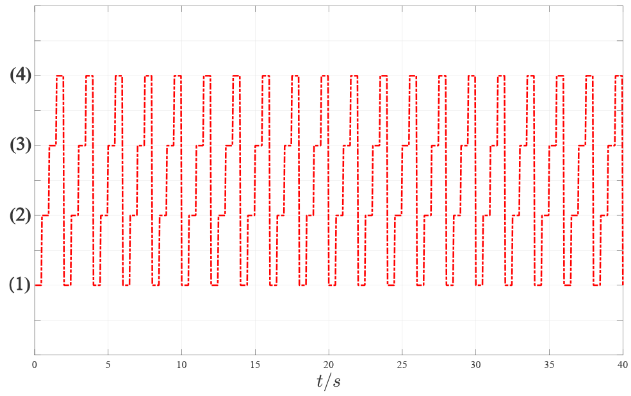


**Fig. 4** *Corresponding topology number at different time*

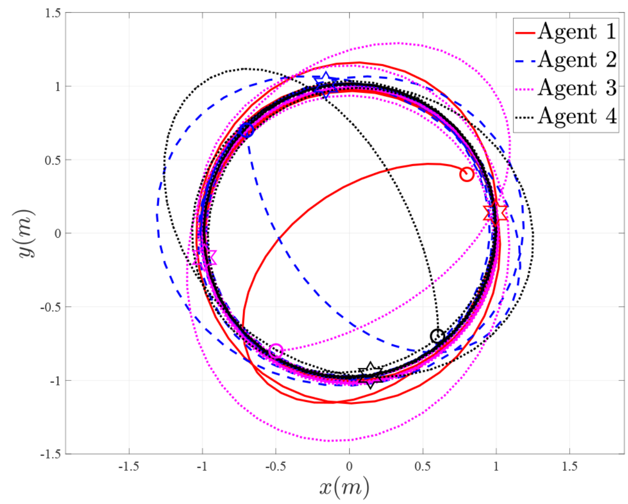


**Fig. 5** *Position trajectories of four agents within 40 s in the simulation*

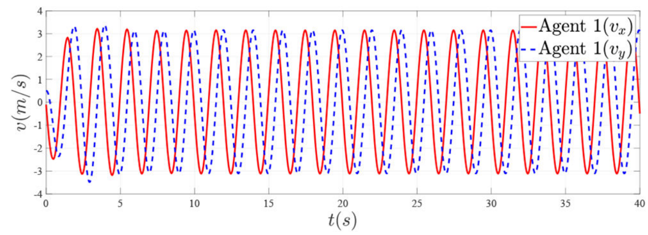


**Fig. 6** *Velocity v(k) of agent 1 within 40 s in the simulation*

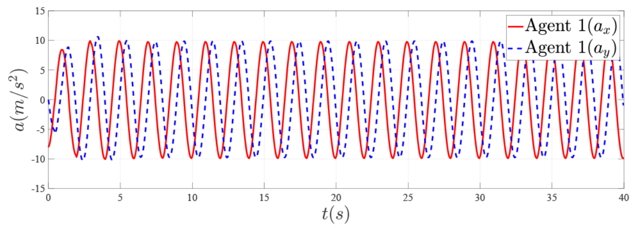


**Fig. 7** *Control input u(k) of agent 1 within 40 s in the simulation*

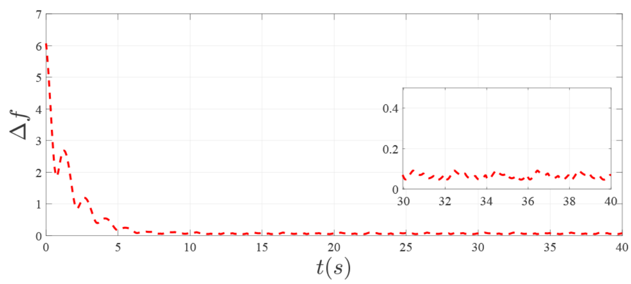


**Fig. 8** *Formation error $\Delta f(k)$ within 40 s in the simulation*

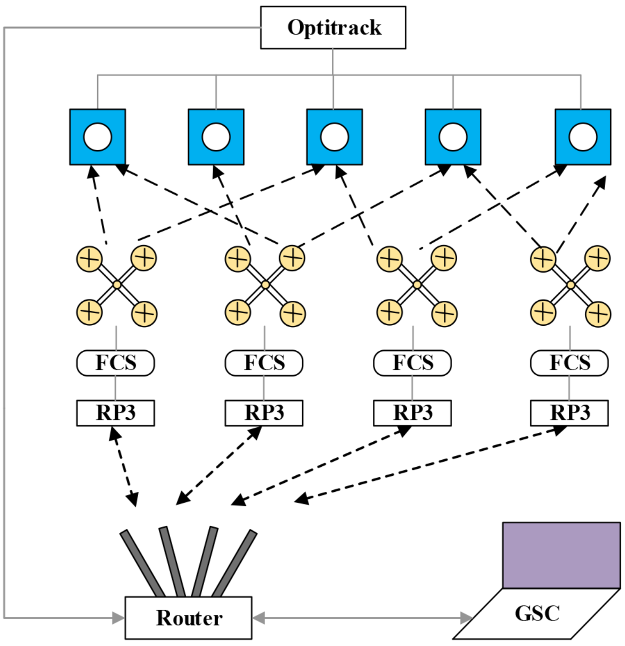


**Fig. 9** *Structure of UAV formation platform*

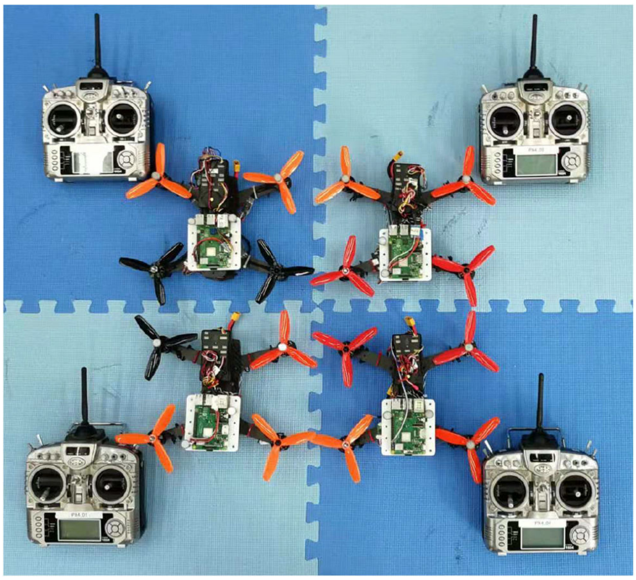

**Fig. 10** *Quadrotor UAVs and remote controls used in the experiments*

GSC. Each quadrotor is directly controlled by the on-board RP3, which can communicate with each other via the router. The control rate of RP3 is 20 hZ and the indoor positioning system sends the state information of each quadrotor to the GSC every 0.05 s. The fly data is saved in the matlab workspace by GSC, which can be analysed after the experiments. However, in order to deal with the emergencies, each quadrotor is equipped with a remote control receiver (RC receiver). In any emergency cases, the pilot can manually control the quadrotor via the remote control. Fig. 10 demonstrates the quadrotor UAVs and remote control used in the experiments. More detailed information about the numbers and key features of the main components of the UAV formation platform can be found in Table 1.

### 5.2 Quadrotor UAV modelling

During the modelling of the quadrotor UAV, two dynamics, namely, attitude dynamics and trajectory dynamics are established. The control constants of attitude dynamics are much smaller than the ones of trajectory dynamics [56], which means that the stability of the attitude control can be reached much faster than that of the trajectory control. Then, the attitude control loop and trajectory control loop of quadrotor can be decoupled and they are called inner-loop control and outer-loop control, respectively. In our UAV formation platform, the attitude controller and trajectory controller are on the RP3 and FCS, respectively. Fig. 11 shows the details of two loops control scheme for the multiple UAVs system.

**Table 1** Main components used in the UAV formation platform

| Components | Numbers | Key features |
|---|---|---|
| indoor positioning system | Optitrack Prime13 | number of camera: 12; range: 12 m; resolution: 0.1–1 mm |
| flight control | PX4 | |
| frame | QAV 250 | diameter: 250 mm; weight: 132 g |
| electronic speed controller | HOBBYWING 20A | rated current: 20 A; weight: 19 g; refresh rate: 50–432 Hz |
| motor | MT2204 | KV: 2300 RPM/V; weight: 27 g; diameter: 27.9 mm |
| propeller | 5045 V3 | diameter: 5 inch; thread pitch: 4.5 inch |
| battery | 2200 mAh-25C 11.1V-3S1P | voltage: 3S/11.1 V; capacity: 2200 mAh; weight: 184 g. |

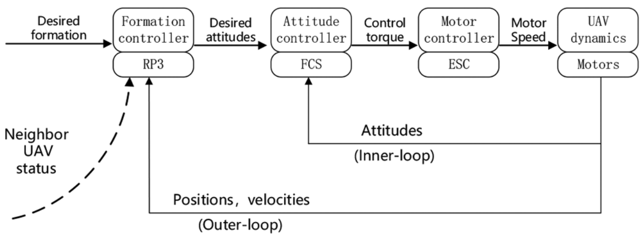


**Fig. 11** *Two-loops control scheme for the multiple UAVs system*

The formation problem discussed in this paper is mainly related to the positions and velocities of the quadrotor UAV, which means that the formation controller acts on the outer-loop in the two-loops control scheme. In this configuration, the trajectory dynamics of the quadrotor UAV can be modelled as a second-order system [57, 58], which can be discretised as (1). Then the discrete-time-varying formation protocol proposed in this paper can be applied to the formation control problem of multiple UAVs system.

### 5.3 Experiment configuration

The multiple UAVs system in the experiment consists of four quadrotor UAVs moving in the horizontal plane. The height and yaw angle of each UAV are controlled to be fixed values 1.2 m and 0°, respectively. The control of UAV in the $X$ and $Y$ directions is decoupled, namely, the controllers in the two directions can be designed separately. Considering the symmetric structure of the quadrotor UAV in this experiment, the controller coefficients in the $X$ and $Y$ directions are set to be the same.

A PD controller is used in the height control with 0.5 and 0.2 as the coefficients of proportional and derivative terms, respectively. A proportional controller with the coefficient 0.9 is used in the yaw angle control. Control protocol (5) is used for trajectory control in the directions $X$ and $Y$. The position and velocity information of the UAVs in the directions $X$ and $Y$ are both taken into consideration. Using Kronecker product to combine the states of UAV in the $X$ and $Y$ directions, denote $x_i(k) = [p_{ix}(k), v_{ix}(k), p_{iy}(k), v_{iy}(k)]^{\mathrm{T}}$, $i \in \{1, 2, 3, 4\}$, where $p_{ix}(k)$ and $v_{ix}(k)$ are the position and velocity

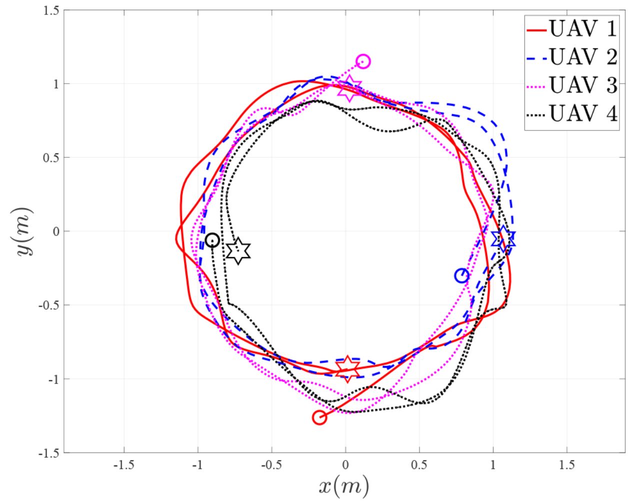


**Fig. 12** *Position trajectories of four UAVs within 40 s in experiment 1*

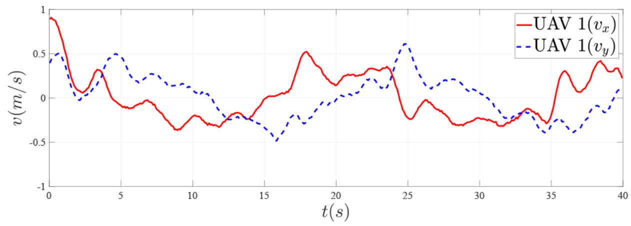


**Fig. 13** *Velocity v(k) of UAV 1 within 40 s in experiment 1*

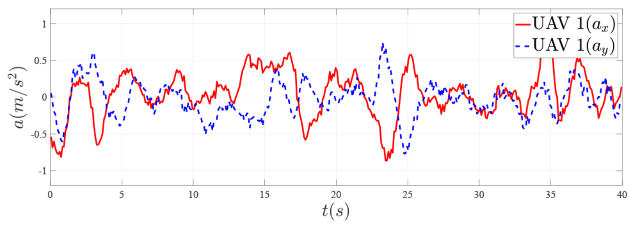


**Fig. 14** *Control input u(k) of UAV 1 within 40 s in experiment 1*

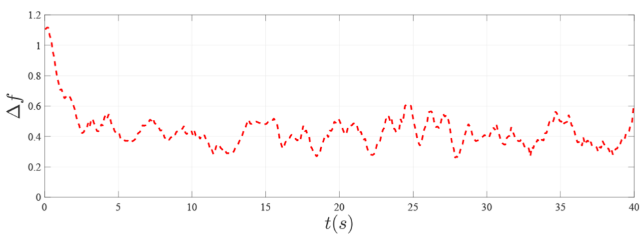


**Fig. 15** *Formation error* $\Delta f(k)$ *within 40 s in experiment 1*

of UAV $i$ in the direction $X$, respectively; $p_{iy}(k)$ and $v_{iy}(k)$ are the position and velocity of UAV $i$ in the direction $Y$, respectively.

Choose the same setting as in simulation. The parameters in protocol (5) are selected as $\bar{k}_{11} = -5$, $\bar{k}_{12} = -1$, $\bar{k}_{21} = 0.2$, and $\bar{k}_{22} = 0.4$. The expected time-varying formation of the multiple UAVs system is a circular motion, where $r = 1$ m, $\omega = 3.14$ rad/s, and the phase difference among four UAVs is $\pi/2$. The sample time $T$ in the experiment is 0.05 s. The topology switching settings are the same as in the simulation. The four possible topologies and the corresponding topology at different times are shown in Figs. 3 and 4, respectively.

The state information of the four UAVs is transmitted via the WIFI-router from the indoor positioning system to the onboard RP3. Testing by *ping* command during the experiment, one can observe that the time-delay between the indoor positioning system and RP3 is around 5 ms. Considering that the additional time-delays among the UAVs are between 50 and 150 ms, the time-delay between the indoor positioning system and RP3 is negligible in comparison with the additional time-delays among the UAVs. Therefore, we assume that the additional time-delays are the only time-delay in the multiple UAVs system.

Two experiments are introduced next. In experiment 1, only the switching topology constraint is considered. In experiment 2, both the switching topology and non-uniform time-delay constraints are taken into consideration.

*5.3.1 Experiment 1:* In this experiment, only the communication topology is switching. In order to avoid collisions when the quadrotor UAVs start to realise the formation, the initial states of four UAVs are selected near the formation trajectory. The initial positions and velocities of the four UAVs are chosen as

$$x_1(0) = [-0.18\,\text{m}, 0.89\,\text{m/s}, -1.26\,\text{m}, 0.39\,\text{m/s}]^{\text{T}}$$
$$x_2(0) = [0.79\,\text{m}, 0.17\,\text{m/s}, -0.30\,\text{m}, 0.41\,\text{m/s}]^{\text{T}}$$
$$x_3(0) = [0.12\,\text{m}, -0.73\,\text{m/s}, 1.15\,\text{m}, -0.33\,\text{m/s}]^{\text{T}}$$
$$x_4(0) = [-0.90\,\text{m}, 0.04\,\text{m/s}, -0.06\,\text{m}, 0.70\,\text{m/s}]^{\text{T}}$$

Fig. 12 shows the positions of the four UAVs in the $X$–$Y$ plane within 40 s, where the initial positions of UAVs are represented by round markers and their positions at 40 s are noted by the hexagon markers. The velocities and control inputs of UAV 1 in the directions $X$ and $Y$ are indicated in Figs. 13 and 14, respectively. The velocities and control inputs of the other 3 UAVs are similar to those of UAV 1. The formation error $\Delta f(k)$ is shown in Fig. 15.

From the position trajectories of four UAVs in Fig. 12, one can obtain that four UAVs achieve a circle formation with radius $r = 1$ m. Taking the UAV 1 as an example, the velocities and control inputs in two directions change periodically with an amplitude 0.5 m/s and 0.8m/s$^2$, respectively. In addition, from Fig. 15, one can see that the formation error $\Delta f(k)$ decreases rapidly at the beginning of the experiment, and it oscillates around 0.4 after 5 s. As a result, the four UAVs systems with the switching topology form the desired circular time-varying formation.

*5.3.2 Experiment 2:* In this experiment, both the switching topology and non-uniform communication time-delay are taken into account. The time-delays among the four UAVs are the same as in the simulation. The initial states of the four UAVs are set to

$$x_1(0) = [0.83\,\text{m}, 0.16\,\text{m/s}, -0.50\,\text{m}, 1.00\,\text{m/s}]^{\text{T}}$$
$$x_2(0) = [0.52\,\text{m}, -0.25\,\text{m/s}, 0.73\,\text{m}, 0.23\,\text{m/s}]^{\text{T}}$$
$$x_3(0) = [-0.96\,\text{m}, -0.11\,\text{m/s}, 0.70\,\text{m}, -0.77\,\text{m/s}]^{\text{T}}$$
$$x_4(0) = [-0.57\,\text{m}, 0.31\,\text{m/s}, -0.97\,\text{m}, -0.33\,\text{m/s}]^{\text{T}}$$

Fig. 16 shows the positions of the four UAVs in the $X$–$Y$ plane within 40 s, where the initial positions of UAVs are represented by round markers and their positions at 40 s are noted by the hexagon markers. The velocities and control inputs of UAV 1 in the directions $X$ and $Y$ are indicated in Figs. 17 and 18, respectively. The velocities and control inputs of the other three UAVs are similar to those of UAV 1. The formation error $\Delta f(k)$ is shown in Fig. 19.

From the position trajectories of four UAVs in Fig. 16, one can obtain that four UAVs achieve a circle formation with radius $r = 1$ m. Taking the UAV 1 as an example, the velocities and control inputs in two directions change periodically with amplitude 0.5 m/s and 0.8 m/s$^2$, respectively. In addition, from Fig. 19, one can see that the formation error $\Delta f(k)$ decreases rapidly at the beginning of the experiment, and it oscillates around 0.4 after 3 s. As a result, the four UAVs systems with the switching topology and non-uniform communication time-delay form the desired circular time-varying formation (Fig. 20).

## 6 Conclusion

Time-varying formation control problem for second-order discrete-time multi-agent systems with communication delays and

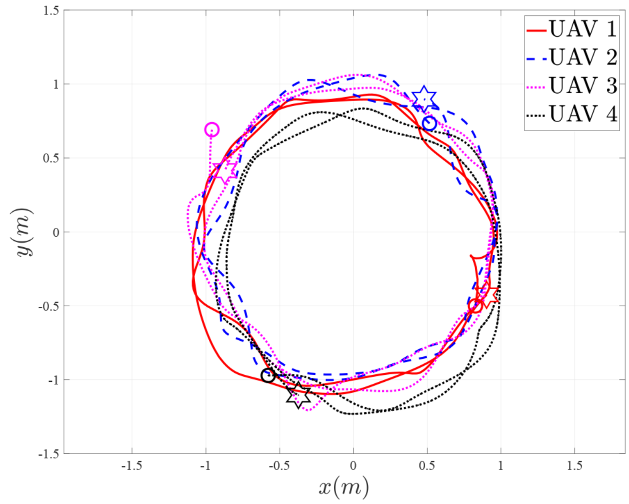


**Fig. 16** *Position trajectories of four UAVs within 40 s in experiment 2*

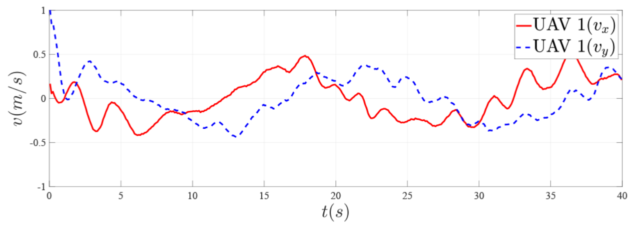


**Fig. 17** *Velocity v(k) of UAV 1 within 40 s in experiment 2*

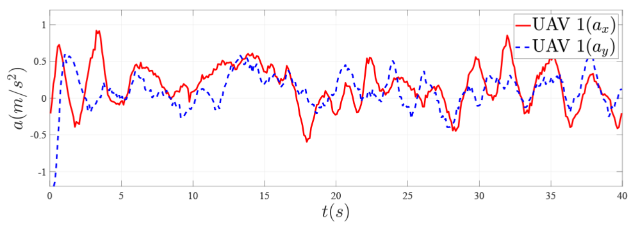


**Fig. 18** *Control input u(k) of UAV 1 within 40 s in experiment 2*

switching topology was studied in this paper, where the communication delays were assumed to be time-varying and non-uniform. Using the real-time and time-delayed agents' state information, a discretised formation protocol was established. The formation feasibility condition that considered the properties of the desired formation vector was given. Through an approach of state transformation, sufficient conditions for the realisation of the time-varying formation of second-order discrete-time multi-agent systems were presented. Simulation verified the stability of the closed-loop system with the formation protocol. Based on a UAV formation flight platform, two experiments with four quadrotor UAVs were carried out. Experiment 1 showed that the desired time-varying formation with switching topology can be realised and Experiment 2 demonstrated that the additional non-uniform communication delays with maximum 0.15 s had a very small impact on the formation, which meant that the proposed protocol was efficient on the formation control problem of UAVs. Therefore, the proposed formation protocol can be applied to the time-varying formation control problem of multiple UAVs system with non-uniform communication delays and topology switching.

## 7 Acknowledgments

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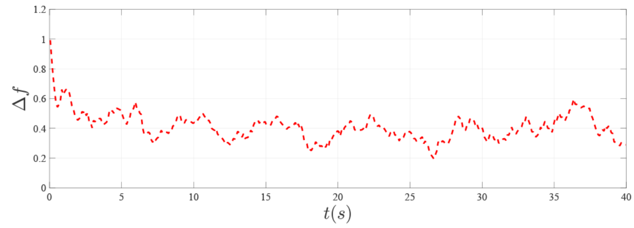


**Fig. 19** *Formation error $\Delta f(k)$ within 40 s in experiment 2*

**Fig. 20** *Four UAVs during the formation*

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