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Study of Barankin bound vs Cramér-Rao bound for interferometric-like array design at low SNR

Jianhua Wang [a,*], Lucien Bacharach [a], Mohammed Nabil El Korso [b], Pascal Larzabal [a]

[a] SATIE, Université Paris-Saclay, 3 rue Joliot Curie, Bâtiment Breguet, 91190, Gif-sur-Yvette, France
[b] L2S, Université Paris-Saclay, 91190 Gif-sur-Yvette, France

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ABSTRACT

In this paper, we address the antenna array design problem at low signal-to-noise ratio (SNR). The Cramér-Rao bound (CRB) is the most commonly used criterion to solve the array optimization problem due to its computing simplicity and tightness in the asymptotical region. However, there exists a threshold SNR at which the estimation variance significantly deviates from the CRB. In this case, the CRB is no longer a tight bound. To address this issue, we propose the use of the Barankin Bound (BB) on the source location and source intensity in astrometry and photometry problems as an alternative optimization criterion. BB provides a mean square error (MSE)-optimal trade-off mainlobe width and sidelobe level of beampattern. The performance of the obtained array geometries is assessed and compared by evaluating the aforementioned bounds and the mean square error (MSE) on the estimation of source location and intensity. The simulation results illustrate that the BB-based criterion provides a trade-off between increasing the estimation accuracy and reducing the ambiguity.

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* Corresponding author.
E-mail address: jianhua.wang@universite-paris-saclay.fr (J. Wang).

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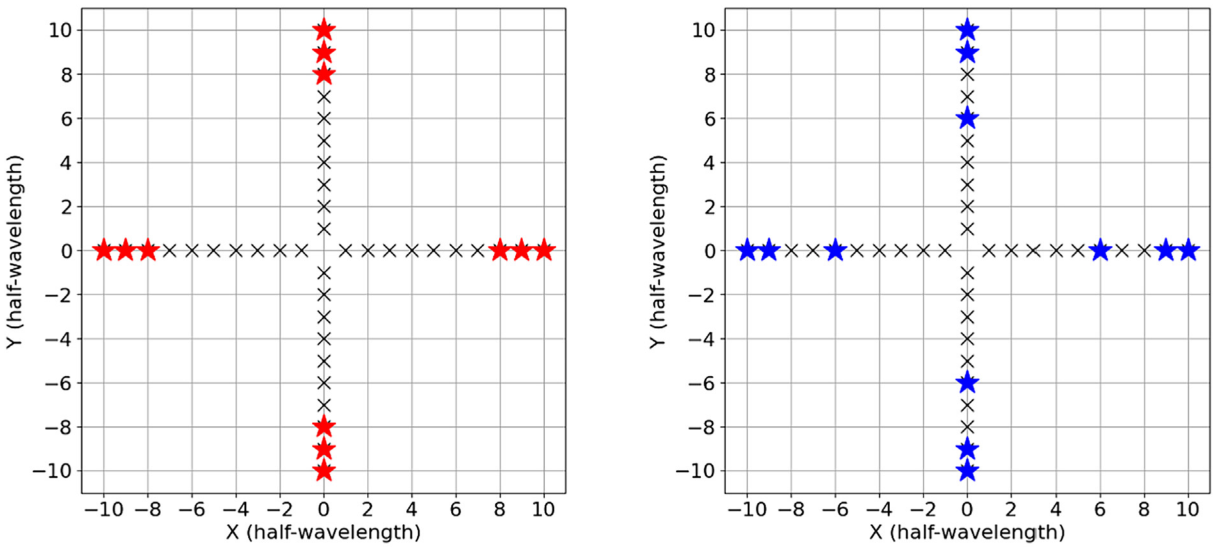
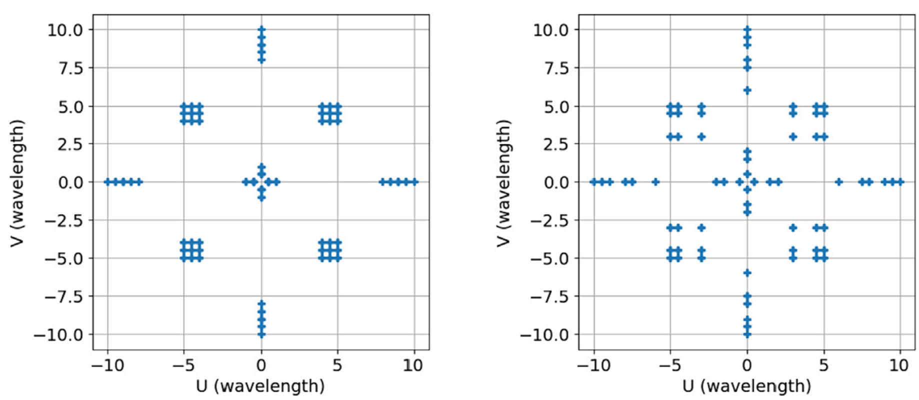
Figures and tables



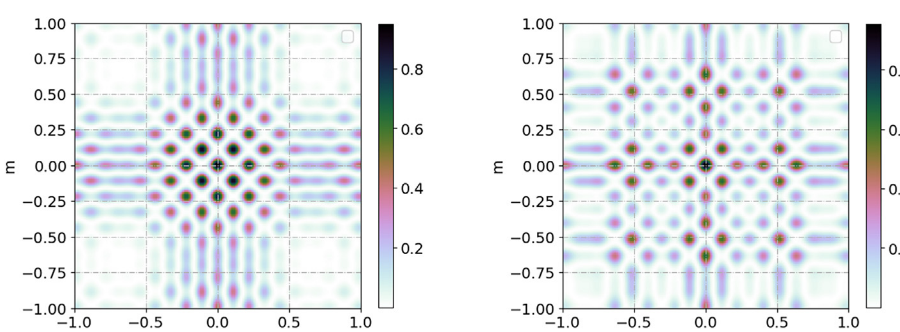
(a) (b)

Fig. 1. The antenna array given by (a) CRB-based criterion, (b) BB-based criterion.



(a) (b)

Fig. 2. The UV plane coverage of the array given by (a) CRB-based criterion, (b) BB-based criterion.



(a) (b)

Fig. 3. (a) The 2D beampattern of the array given by CRB-based criterion. (b) The 2D beampattern of the array given by BB-based criterion.

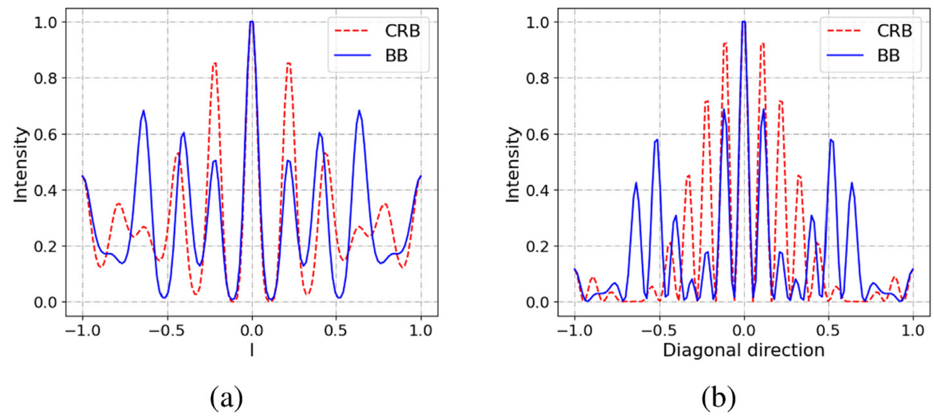


Fig. 4. (a) The 1D beampattern of the l-direction. (b) The 1D beampattern of the diagonal direction. Red dashed (blue solid) lines represent the result of the array geometries given by CRB (BB)-based optimization criteria. Since it is quite difficult to compare directly the 2D beampatterns, two special axes are selected to better illustrate the beampattern in 1D: the l-direction and the diagonal direction (l = m), which are shown in Fig. 4.

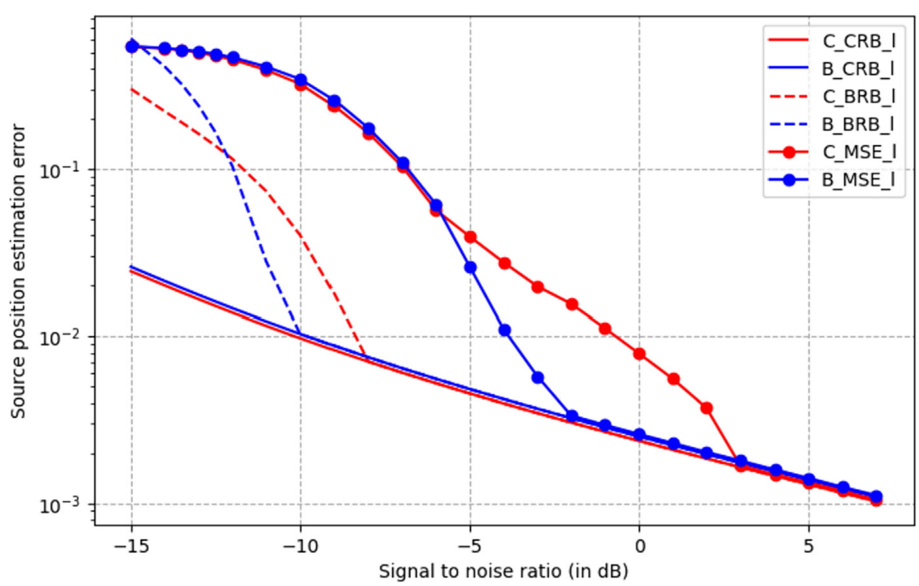


Fig. 5. Bounds and MSEs on l estimation. Solid lines, dashed lines, and solid lines with points markers in red (blue) represent the CRB, BB, and MSEs of the array geometries given by CRB (BB)-based optimization criteria.

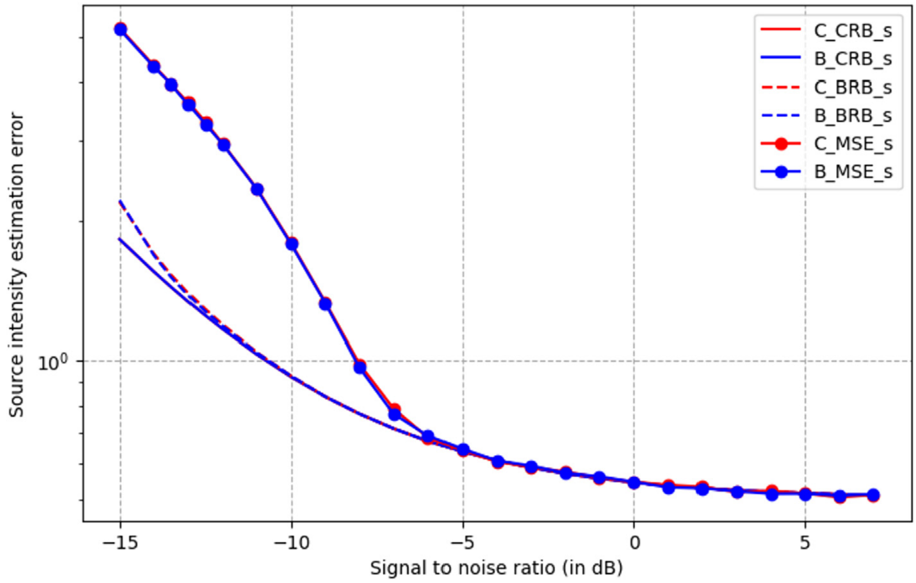


Fig. 6. Bounds and MSE on σ_s^2 estimation. Solid lines, dashed lines, and solid lines with points markers in red (blue) represent the CRB, BB, and MSEs of the array geometries given by CRB (BB)-based optimization criteria.

CRediT authorship contribution statement

Jianhua Wang: Writing – original draft, Software, Methodology, Formal analysis. **Lucien Bacharach:** Writing – review & editing, Methodology, Formal analysis. **Mohammed Nabil El Korso:** Writing – review & editing, Methodology, Formal analysis. **Pascal Larzabal:** Writing – review & editing, Methodology, Formal analysis.

Data availability

Data will be made available on request.

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Declaration of interests

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

Further reading

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[2] NenUFAR, https://nenufar.obs-nancay.fr/ [Online].
[3] Square kilometre array, https://www.skatelescope.org/ [Online].
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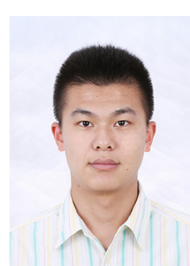

Jianhua Wang received his Master's Degree in Industrial Engineering and his Engineer's degree in Computer Science, Telecommunications and Networks from Centrale Pékin, Beihang University, Beijing, China, in 2021. Since 2021, he has been a Ph.D student in Statistical Signal and Image Processing at SATIE, ENS Paris-Saclay, Paris-Saclay University, France. His primary research interests are in multisensor array processing, antenna array design and image reconstruction in radio astronomy.

Lucien Bacharach received the Graduate degree in aeronautical engineering from ISAE-ENSICA, Toulouse, France, in 2014, and the Master Research degree in signal and image processing in 2015, and the Ph.D. degree in 2018 from Paris-Sud University, Orsay, France. He is currently an Assistant Professor with the Paris-Saclay University. His research interests and activities include estimation theory and performance analysis in statistical signal processing, and antenna array design in radio astronomy.

Mohammed Nabil El Korso obtained the M.Sc. from the National Polytechnic School, Algeria, Master degree and Ph.D. from Paris-Saclay University, and the HDR from Paris Nanterre University. From 2013 to 2022, he was an Assistant Professor at Paris Nanterre University and he is currently Professor at Paris-Saclay University. His research interest lies in statistical methods for signal processing with emphasis on robust and adaptive detection/estimation, asymptotic and non-asymptotic performances analysis and array processing applied to multi-sensor systems, radar, sonar and interferometers in the context of radio-astronomy. He is a member of the EURASIP Special Area Team TMTSP (Theoretical and Methodological Trends in Signal Processing) and Associate Editor for IEEE Transactions on Signal Processing and Elsevier Signal Processing.

Pascal Larzabal received the PhD in Signal processing in 1992. He is currently Professor of Electrical Engineering at University Paris-Saclay. He teaches signal processing and mathematics. His research concerns estimation in array processing for wavefront identification, geolocalization and radioastronomy.